MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SOLUTIONS

1. (a) $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n}\frac{2}{(x_i^*-4)^2}\Delta x_i = \int_6^8 \frac{2}{(x-4)^2}\,dx$

   (b) $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n}\cos^3(5x_i^*)\Delta x_i = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^3(5x)\,dx$

2. (a) Using a regular partition and setting the sample point $x_i^* = x_i$, we can write

$$\int_0^2 \frac{x^3}{4}\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\Delta x.$$

Let

$$\Delta x = \frac{2-0}{n} = \frac{2}{n} \quad \text{and} \quad x_i^* = 0 + \frac{2i}{n} = \frac{2i}{n}$$

so

$$f(x_i^*) = \frac{x_i^3}{4} = \frac{1}{4}\left(\frac{2i}{n}\right)^3 = \frac{2i^3}{n^3}.$$

Then

$$\begin{aligned}\int_0^2 \frac{x^3}{4}\,dx &= \lim_{n\to\infty}\sum_{i=1}^{n}\frac{2i^3}{n^3}\cdot\frac{2}{n}\\ &= \lim_{n\to\infty}\frac{4}{n^4}\sum_{i=1}^{n} i^3\\ &= \lim_{n\to\infty}\frac{4}{n^4}\cdot\frac{n^2(n+1)^2}{4}\\ &= \lim_{n\to\infty}\frac{n^2+2n+1}{n^2}\\ &= 1.\end{aligned}$$

Since $f(x)$ is continuous and non-negative on $[0,2]$ the definite integral represents the area under the curve.

   (b) Using a regular partition and setting the sample point $x_i^* = x_i$, we can observe that

$$\int_2^3 (2-7x)\,dx = \lim_{n\to\infty} f(x_i^*)\Delta x$$

where

$$\Delta x = \frac{3-2}{n} = \frac{1}{n} \quad \text{and} \quad x_i^* = 2 + \frac{i}{n}.$$

Now we have

$$f(x_i^*) = 2 - 7\left(2 + \frac{i}{n}\right) = -\frac{7i}{n} - 12$$

and so

$$\begin{aligned}
\int_2^3 (2-7x)\,dx &= \lim_{n\to\infty} \sum_{i=1}^{n} \left(-\frac{7i}{n} - 12\right) \cdot \frac{1}{n} \\
&= \lim_{n\to\infty} \left(-\frac{7}{n^2} \sum_{i=1}^{n} i - \frac{12}{n} \sum_{i=1}^{n} 1\right) \\
&= \lim_{n\to\infty} \left(-\frac{7}{n^2} \cdot \frac{n(n+1)}{2} - \frac{12}{n} \cdot n\right) \\
&= \lim_{n\to\infty} \left(-\frac{7(n+1)}{2n} - 12\right) \\
&= -\frac{7}{2} - 12 \\
&= -\frac{31}{2}.
\end{aligned}$$

However, note that $2 - 7x < 0$ whenever $x > \frac{2}{7}$, and therefore on $[2, 3]$. This means that the definite integral does not represent the area under the curve. Indeed, it would make no sense to assign a negative value to an area.