MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

TEST 1 | **MATHEMATICS 1001-001** | WINTER 2026

SOLUTIONS

[5] 1. (a) Let $u = x^3$ so $du = 3x^2\,dx$ and $\frac{1}{3}\,du = x^2\,dx$. The integral becomes

$$\begin{aligned}\int x^2\cos(x^3)\,dx &= \frac{1}{3}\int \cos(u)\,du \\ &= \frac{1}{3}\sin(u) + C \\ &= \frac{1}{3}\sin(x^3) + C.\end{aligned}$$

[6] (b) We use integration by parts with $w = x^2$ so $dw = 2x\,dx$, and $dv = \cos(3x)\,dx$ so $v = \frac{1}{3}\sin(3x)$. Thus

$$\int x^2\cos(3x)\,dx = \frac{1}{3}x^2\sin(3x) - \frac{2}{3}\int x\sin(3x)\,dx.$$

Now we use integration by parts again. This time we let $w = x$ so $dw = dx$, and $dv = \sin(3x)\,dx$ so $v = -\frac{1}{3}\cos(3x)$. This yields

$$\begin{aligned}\int x^2\cos(3x)\,dx &= \frac{1}{3}x^2\sin(3x) - \frac{2}{3}\left[-\frac{1}{3}x\cos(3x) + \frac{1}{3}\int\cos(3x)\,dx\right] \\ &= \frac{1}{3}x^2\sin(3x) + \frac{2}{9}x\cos(3x) - \frac{2}{9}\cdot\frac{1}{3}\sin(3x) + C \\ &= \frac{1}{3}x^2\sin(3x) + \frac{2}{9}x\cos(3x) - \frac{2}{27}\sin(3x) + C.\end{aligned}$$

[5] (c) We can rewrite the integral as

$$\begin{aligned}\int\left(\frac{\cos(x)+3}{\cos(x)}\right)^2 dx &= \int\left(\frac{\cos(x)}{\cos(x)} + \frac{3}{\cos(x)}\right)^2 dx \\ &= \int[1 + 3\sec(x)]^2\,dx \\ &= \int[1 + 6\sec(x) + 9\sec^2(x)]\,dx \\ &= x + 6\ln|\sec(x)+\tan(x)| + 9\tan(x) + C.\end{aligned}$$

[12] 2. (a) First we complete the square:

$$\begin{aligned}15 - 4x - 4x^2 = -4\left[x^2 + x - \frac{15}{4}\right]\\ &= -4\left[\left(x^2 + x + \frac{1}{4}\right) - \frac{15}{4} - \frac{1}{4}\right]\\ &= -4\left[\left(x + \frac{1}{2}\right)^2 - 4\right]\\ &= 16 - 4\left(x + \frac{1}{2}\right)^2\\ &= 16 - (2x+1)^2.\end{aligned}$$

Thus

$$\frac{1}{\sqrt{15 - 4x - 4x^2}}\,dx = \int \frac{1}{\sqrt{16 - (2x+1)^2}}\,dx.$$

Now we let $u = 2x + 1$ so $du = 2\,dx$ and $\frac{1}{2}\,du = dx$. The integral becomes

$$\begin{aligned}\frac{1}{\sqrt{15 - 4x - 4x^2}}\,dx &= \frac{1}{2}\int \frac{1}{\sqrt{16 - u^2}}\,du\\ &= \frac{1}{2}\arcsin\left(\frac{u}{4}\right) + C\\ &= \frac{1}{2}\arcsin\left(\frac{2x+1}{4}\right) + C.\end{aligned}$$

(b) We use integration by parts. One approach is to let $w = e^{2x}$ so $dw = 2e^{2x}\,dx$, and $dv = \sin(x)\,dx$ so $v = -\cos(x)$. Then

$$\int e^{2x}\sin(x)\,dx = -e^{2x}\cos(x) + 2\int e^{2x}\cos(x)\,dx.$$

We use integration by parts again. We still let $w = e^{2x}$ so $dw = 2e^{2x}\,dx$, but now we let $dv = \cos(x)\,dx$ so $v = \sin(x)$. Thus

$$\begin{aligned}\int e^{2x}\sin(x)\,dx &= -e^{2x}\cos(x) + 2\left[e^{2x}\sin(x) - 2\int e^{2x}\sin(x)\,dx\right]\\ &= -e^{2x}\cos(x) + 2e^{2x}\sin(x) - 4\int e^{2x}\sin(x)\,dx.\end{aligned}$$

Now observe that the original integral has reoccurred, so we can write

$$\begin{aligned}5\int e^{2x}\sin(x)\,dx &= -e^{2x}\cos(x) + 2e^{2x}\sin(x)\\ \int e^{2x}\sin(x)\,dx &= -\frac{1}{5}e^{2x}\cos(x) + \frac{2}{5}e^{2x}\sin(x) + C.\end{aligned}$$

(c) Observe that we can rewrite the integral as

$$\int x^3\sqrt{x^2+1}\,dx = \int x^2\sqrt{x^2+1}\cdot x\,dx.$$

We let $u = x^2+1$ so $du = 2x\,dx$ and $\frac{1}{2}\,du = x\,dx$. Then $x^2 = u-1$, and the integral becomes

$$\begin{aligned}\int x^3\sqrt{x^2+1}\,dx &= \int (u-1)\sqrt{u}\cdot\frac{1}{2}\,du \\ &= \frac{1}{2}\int (u^{\frac{3}{2}} - u^{\frac{1}{2}})\,du \\ &= \frac{1}{2}\left[\frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}}\right] + C \\ &= \frac{1}{5}(x^2+1)^{\frac{5}{2}} - \frac{1}{3}(x^2+1)^{\frac{3}{2}} + C.\end{aligned}$$

[12] 3. (a) Observe that we can rewrite the integral as

$$\int \frac{\sec(x)\tan(x)}{9\sec^2(x)+25}\,dx = \int \frac{\sec(x)\tan(x)}{[3\sec(x)]^2+25}\,dx.$$

Let $u = 3\sec(x)$ so $du = 3\sec(x)\tan(x)\,dx$ and $\frac{1}{3}\,du = \sec(x)\tan(x)\,dx$. The integral becomes

$$\begin{aligned}\int \frac{\sec(x)\tan(x)}{9\sec^2(x)+25}\,dx &= \frac{1}{3}\int \frac{1}{u^2+25}\,du \\ &= \frac{1}{3}\cdot\frac{1}{5}\arctan\left(\frac{u}{5}\right) + C \\ &= \frac{1}{15}\arctan\left(\frac{3\sec(x)}{5}\right) + C.\end{aligned}$$

(b) Since the integrand is an improper rational functions, we use long division of polynomials:

$$\begin{array}{r} x^2 - 3 \\ 2x+1\,\overline{)\,2x^3 + x^2 - 6x} \\ \underline{2x^3 + x^2 \phantom{{}-6x}} \\ -6x \\ \underline{-6x - 3} \\ 3 \end{array}$$

Thus we can write the integral as

$$\int \frac{2x^3 + x^2 - 6x}{2x+1}\,dx = \int \left(x^2 - 3 + \frac{3}{2x+1}\right)\,dx$$

$$= \frac{x^3}{3} - 3x + 3 \cdot \frac{1}{2}\ln|2x+1| + C$$

$$= \frac{1}{3}x^3 - 3x + \frac{3}{2}\ln|2x+1| + C.$$

(c) We use integration by parts with $w = \operatorname{arcsec}(x)$ so $dw = \dfrac{1}{x\sqrt{x^2-1}}\,dx$, and $dv = x\,dx$ so $v = \frac{1}{2}x^2$. This yields

$$\int x\operatorname{arcsec}(x)\,dx = \frac{1}{2}x^2\operatorname{arcsec}(x) - \frac{1}{2}\int \frac{x^2}{x\sqrt{x^2-1}}\,dx$$

$$= \frac{1}{2}x^2\operatorname{arcsec}(x) - \frac{1}{2}\int \frac{x}{\sqrt{x^2-1}}\,dx.$$

For the remaining integral, let $u = x^2 - 1$ so $du = 2x\,dx$ and $\frac{1}{2}\,du = x\,dx$. We obtain

$$\int x\operatorname{arcsec}(x)\,dx = \frac{1}{2}x^2\operatorname{arcsec}(x) - \frac{1}{2}\cdot\frac{1}{2}\int \frac{1}{\sqrt{u}}\,du$$

$$= \frac{1}{2}x^2\operatorname{arcsec}(x) - \frac{1}{4}\int u^{-\frac{1}{2}}\,du$$

$$= \frac{1}{2}x^2\operatorname{arcsec}(x) - \frac{1}{4}[2\sqrt{u}] + C$$

$$= \frac{1}{2}x^2\operatorname{arcsec}(x) - \frac{1}{2}\sqrt{x^2-1} + C.$$