MEMORIAL UNIVERSITY OF NEWFOUNDLAND
DEPARTMENT OF MATHEMATICS AND STATISTICS

SOLUTIONS

1. (a) Let $u = 1 - x^3$ so

$$du = -3x^2\,dx \quad \Longrightarrow \quad -\frac{1}{3}\,du = x^2\,dx.$$

The integral becomes

$$\int x^2(1-x^3)^6\,dx = -\frac{1}{3}\int u^6\,du = -\frac{1}{3}\left[\frac{1}{7}u^7\right] + C = -\frac{1}{21}(1-x^3)^7 + C.$$

(b) Let $u = 4t^2 + 5$ so

$$du = 8t\,dt \quad \Longrightarrow \quad \frac{1}{8}\,du = t\,dt.$$

The integral becomes

$$\int \frac{t}{\sqrt[3]{4t^2+5}}\,dt = \frac{1}{8}\int \frac{du}{\sqrt[3]{u}} = \frac{1}{8}\int u^{-\frac{1}{3}}\,du = \frac{1}{8}\left[\frac{3}{2}u^{\frac{2}{3}}\right] + C = \frac{3}{16}(4t^2+5)^{\frac{2}{3}} + C.$$

(c) Let $u = e^x + 3$ so $du = e^x\,dx$. The integral becomes

$$\int \frac{e^x}{e^x+3}\,dx = \int \frac{du}{u} = \ln|u| + C = \ln|e^x+3| + C.$$

(d) Let $u = \cot(x)$ so

$$du = -\csc^2(x)\,dx \quad \Longrightarrow \quad -\,du = \csc^2(x)\,dx.$$

The integral becomes

$$\int \frac{1}{2}\cot^3(x)\csc^2(x)\,dx = -\frac{1}{2}\int u^3\,du = -\frac{1}{2}\left[\frac{1}{4}u^4\right] + C = -\frac{1}{8}\cot^4(x) + C.$$

(e) Let $u = \sin(4\theta^2)$ so

$$du = 8\theta\cos(4\theta^2)\,d\theta \quad \Longrightarrow \quad \frac{1}{8}\,d\theta = \theta\cos(4\theta^2)\,d\theta.$$

The integral becomes

$$\int \theta\sin(4\theta^2)\cos(4\theta^2)\,d\theta = \frac{1}{8}\int u\,du = \frac{1}{8}\left[\frac{1}{2}u^2\right] + C = \frac{1}{16}\sin^2(4\theta^2) + C.$$

(f) Let $u = \sqrt{x}$ so

$$du = \frac{1}{2\sqrt{x}}\,dx \quad \Longrightarrow \quad 2\,du = \frac{1}{\sqrt{x}}\,dx.$$

The integral becomes

$$\int \frac{\cot\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\int \cot(u)\,du = 2\ln|\sin(u)| + C = 2\ln|\sin(\sqrt{x})| + C.$$

(g) Let $u = x^3 - x$ so $du = 3x^2 - 1$. The integral becomes

$$\int \frac{6x^2 - 2}{x^3 - x}\,dx = \int \frac{2(3x^2 - 1)}{x^3 - x}\,dx = 2\int \frac{du}{u} = 2\ln|u| + C = 2\ln|x^3 - x| + C.$$

(h) Using long division of polynomials, we have

$$\begin{array}{r l}
 & \,x^2 + 2x + 1 \\
x - 2 & \overline{)\,x^3 \qquad\quad - 3x + 1} \\
 & \underline{x^3 - 2x^2} \\
 & \qquad 2x^2 - 3x + 1 \\
 & \qquad \underline{2x^2 - 4x} \\
 & \qquad\qquad\quad x + 1 \\
 & \qquad\qquad\quad \underline{x - 2} \\
 & \qquad\qquad\qquad\; 3
\end{array}$$

so

$$\frac{x^3 - 3x + 1}{x - 2} = x^2 + 2x + 1 + \frac{3}{x - 2}.$$

Then

$$\int \frac{x^3 - 3x + 1}{x - 2}\,dx = \frac{1}{3}x^3 + x^2 + x + 3\ln|x - 2| + C.$$

(i) Let $u = \cos^2(x)$ so

$$du = -2\sin(x)\cos(x)\,dx \quad \Longrightarrow \quad -\frac{1}{2}\,du = \sin(x)\cos(x)\,dx.$$

The integral becomes

$$\int e^{\cos^2(x)}\sin(x)\cos(x)\,dx = -\frac{1}{2}\int e^u\,du = -\frac{1}{2}e^u + C = -\frac{1}{2}e^{\cos^2(x)} + C.$$

(j) Let $u = \ln(x)$ so $du = \frac{dx}{x}$. The integral becomes

$$\int \frac{1}{x\sqrt{\ln(x)}}\,dx = \frac{1}{\sqrt{u}}\,du = \int u^{-\frac{1}{2}}\,du = 2u^{\frac{1}{2}} + C = 2\sqrt{\ln(x)} + C.$$

(k) Let $u = 5 - x$ so
$$du = -\,dx \quad\Longrightarrow\quad -\,du = dx.$$
Also, $x = 5 - u$ so $x - 2 = 3 - u$. The integral becomes
$$\begin{aligned}\int (x-2)\sqrt{5-x}\,dx &= -\int (3-u)\sqrt{u}\,du \\ &= -3\int u^{\frac{1}{2}}\,du + \int u^{\frac{3}{2}}\,du \\ &= -3\left[\frac{2}{3}u^{\frac{3}{2}}\right] + \frac{2}{5}u^{\frac{5}{2}} + C \\ &= -2(5-x)^{\frac{3}{2}} + \frac{2}{5}(5-x)^{\frac{5}{2}} + C.\end{aligned}$$

(ℓ) Let $u = x^2 + 8$ so
$$du = 2x\,dx \quad\Longrightarrow\quad \frac{1}{2}\,du = x\,dx.$$
Also note that $x^2 = u - 8$. The integral becomes
$$\begin{aligned}\int x^3(x^2+8)^4\,dx &= \int x^2(x^2+8)^4 x\,dx \\ &= \frac{1}{2}\int (u-8)u^4\,du \\ &= \frac{1}{2}\int u^5\,du - 4\int u^4\,du \\ &= \frac{1}{2}\left[\frac{1}{6}u^6\right] - 4\left[\frac{1}{5}u^5\right] + C \\ &= \frac{1}{12}(x^2+8)^6 - \frac{4}{5}(x^2+8)^5 + C.\end{aligned}$$