MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SOLUTIONS

1. (a) $\int 10(5x+4)^8\,dx = 10\int (5x+4)^8\,dx = 10\cdot\frac{(5x+4)^9}{(5)(9)} + C = \frac{2}{9}(5x+4)^9 + C$

   (b) $\int e^{\left(6-\frac{x}{2}\right)}\,dx = \frac{1}{-\frac{1}{2}}e^{\left(6-\frac{x}{2}\right)} + C = -2e^{\left(6-\frac{x}{2}\right)} + C$

   (c) $$\int \frac{t^5+3t^3+\sqrt{t}}{9t^4}\,dt = \int\left(\frac{t}{9}+\frac{1}{3t}+\frac{1}{9}t^{-\frac{7}{2}}\right)dt = \frac{t^2}{18}+\frac{1}{3}\ln|t| + \frac{1}{9}\left(\frac{t^{-\frac{5}{2}}}{-\frac{5}{2}}\right) + C$$
   $$= \frac{t^2}{18}+\frac{1}{3}\ln|t| - \frac{2}{45}t^{-\frac{5}{2}} + C$$

   (d) $$\int\left[\sec^2(4x-1)+\sqrt{4x-1}\right]dx = \frac{1}{4}\tan(4x-1) + \frac{(4x-1)^{\frac{3}{2}}}{(4)\left(\frac{3}{2}\right)} + C$$
   $$= \frac{1}{4}\tan(4x-1) + \frac{1}{6}(4x-1)^{\frac{3}{2}} + C$$

   (e) $$\int \csc(\theta)[\csc(\theta)-\cot(\theta)]\,d\theta = \int[\csc^2(\theta)-\csc(\theta)\cot(\theta)]\,d\theta$$
   $$= -\cot(\theta)+\csc(\theta)+C$$

   (f) Recall that $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$, so we can rewrite the given integral as
   $$\int 4\sin(-4x)\cos(-4x)\,dx = 2\int\sin(-8x)\,dx = -2\frac{\cos(-8x)}{-8} + C$$
   $$= \frac{1}{4}\cos(-8x) + C.$$

   (g) We begin by expanding the numerator, and then dividing through by the denominator:
   $$\int \frac{\left(1+\frac{1}{x}\right)\left(1+\frac{4}{x}\right)}{3x}\,dx = \int\frac{\left(1+\frac{5}{x}+\frac{4}{x^2}\right)}{3x}\,dx = \int\left(\frac{1}{3x}+\frac{5}{3}x^{-2}+\frac{4}{3}x^{-3}\right)dx$$
   $$= \frac{1}{3}\ln|x| - \frac{5}{3}x^{-1} + \frac{4}{3}\cdot\frac{x^{-2}}{-2} + C$$
   $$= \frac{1}{3}\ln|x| - \frac{5}{3x} - \frac{2}{3x^2} + C.$$

2. Note that $F'(x) = e^{\cos(x)} - x^3$ so $F'(0) = e^1 - 0 = e$.

3. $g(x) = \frac{d}{dx}[e^{\cos(x)} - x^3 + C] = e^{\cos(x)}\cdot\frac{d}{dx}[\cos(x)] - 3x^2 + 0 = -\sin(x)e^{\cos(x)} - 3x^2$