

MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 4.5

Math 1001 Worksheet

WINTER 2024

For practice only. Not to be submitted.

1. Determine whether each of the following functions is a probability density function.

$$(a) f(x) = \begin{cases} \frac{x-1}{(x+4)^3}, & \text{for } 0 \leq x \leq 8 \\ 0, & \text{otherwise} \end{cases}$$

$$(b) f(x) = \begin{cases} \frac{1}{(x+4)^3}, & \text{for } 0 \leq x \leq 8 \\ 0, & \text{otherwise} \end{cases}$$

$$(c) f(x) = \begin{cases} \frac{18x}{(x+4)^3}, & \text{for } 0 \leq x \leq 8 \\ 0, & \text{otherwise} \end{cases}$$

2. Consider the function $f(x) = \frac{k}{x^2 + 1}$.

- (a) Determine the value of the constant k which would ensure that $f(x)$ is a probability density function.
- (b) Calculate $P(-\sqrt{3} \leq X \leq \sqrt{3})$.

3. Consider the function

$$f(x) = \begin{cases} kxe^{-2x}, & \text{for } x \geq 0 \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Determine the value of the constant k which would ensure that $f(x)$ is a probability density function.
- (b) Calculate $P(0 \leq X \leq 2)$.

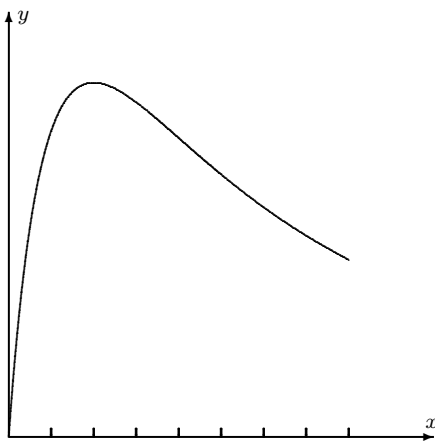


Figure 1: The probability density function considered in Question 4.

4. Consider the scenario in which we choose two random videos from the search of history of our favourite video-sharing platform, and add together the viewing time from the first 4 minutes of each, giving possible outcomes of between 0 and 8 minutes. The probability density function identified in Question 1(c) is depicted in Figure 1, and represents a more realistic model for typical human behaviour than the parabolic probability density function that we considered in class. In particular, it assumes that viewers are likely to watch at least the first few seconds of most of the videos for which they search. Use it to determine each of the following.
 - (a) The probability that the total viewing time of two randomly-chosen videos is between 3 minutes and 5 minutes.
 - (b) The probability that the total viewing time of two randomly-chosen videos is less than 1 minute.
 - (c) The probability that the total viewing time of two randomly-chosen videos is greater than 6 minutes.
 - (d) The mean viewing total viewing time μ of all pairs of videos, to one decimal place.
5. Ruby is trying to write a computer program which will output a random real number between 0 and 5.
 - (a) Such a program should be described by the probability density function

$$f(x) = \begin{cases} \frac{1}{5}, & \text{for } 0 \leq x \leq 5 \\ 0, & \text{otherwise.} \end{cases}$$

Show that this is, in fact, a probability density function and find its mean value μ .

- (b) Unfortunately, an error in Ruby's code means that the probability of outputting a real number between 4 and 5 is twice as great as the probability of outputting a real number between 0 and 4. Determine a probability density function which describes this situation, and find its mean value μ .