SECTION 4.2 **Math 1001 Worksheet** WINTER 2025

For practice only. Not to be submitted.

1. Solve each of the following separable differential equations.

   (a) $\dfrac{t}{y^2+1}\dfrac{dy}{dt} + \dfrac{t^2}{e^t} = 0$

   (b) $\dfrac{5\cos(t)}{y+2}\dfrac{dy}{dt} = \dfrac{y-3}{\csc(t)}$

2. Solve the initial value problem
$$t\frac{dy}{dt} - \sqrt{4-y^2} = 0, \quad y(1) = 2.$$

3. The half-life of Einsteinium-254 is 270 days. A sample initially has a mass of 3 mg.

   (a) How much is left in the sample after 30 days?

   (b) After how many days will the sample be reduced to 0.5 mg?

4. A group of "castaways" arrives on a deserted island for a reality game show. While there, they learn that a number of parakeets were relocated to the island 2 years before; this population now numbers roughly 50 birds. Three years later, some of the "castaways" return to the island for an "all-star" edition of the show. They discover that there are now about 150 parakeets. If the population has been growing at a rate proportional to its size, how many parakeets were there originally?

5. A flu virus passes through the people living in a city at an exponential rate. If 10% of the population is infected after 10 days, how long will it take for 40% of the people to contract the flu?

6. Newton's Law of Cooling states that the rate of change of the temperature of a warm object in a cool environment is proportional not to the temperature of the object, but to the *difference* in the temperature between the object and the environment.

   (a) Let y be the temperature of the object, y_0 be its initial temperature, T be the temperature of the environment, and k be the constant of proportionality. Identify a differential equation to model Newton's Law of Cooling and find its solution.

   (b) Foul play befalls a math professor who failed one too many students. The police discover his body (which has cooled from 37°C to 25°C) half an hour after his demise, on a MUN parking lot where the temperature is −8°C. The medical examiner arrives on the scene 15 minutes after the police. Use Newton's Law of Cooling to find the temperature of the math prof's body at this time.