

MEMORIAL UNIVERSITY OF NEWFOUNDLAND
DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 4.2

Math 1001 Worksheet

WINTER 2024

For practice only. Not to be submitted.

1. Solve each of the following separable differential equations.

(a) $\frac{t}{y^2 + 1} \frac{dy}{dt} + \frac{t^2}{e^t} = 0$

(b) $\frac{5 \cos(t) dy}{y + 2} = \frac{y - 3}{\csc(t)}$

2. Solve the initial value problem

$$t \frac{dy}{dt} - \sqrt{4 - y^2} = 0, \quad y(1) = 2.$$

3. The half-life of Einsteinium-254 is 270 days. A sample initially has a mass of 3 mg.

- (a) How much is left in the sample after 30 days?
(b) After how many days will the sample be reduced to 0.5 mg?

4. A group of “castaways” arrives on a deserted island for a reality game show. While there, they learn that a number of parakeets were relocated to the island 2 years before; this population now numbers roughly 50 birds. Three years later, some of the “castaways” return to the island for an “all-star” edition of the show. They discover that there are now about 150 parakeets. If the population has been growing at a rate proportional to its size, how many parakeets were there originally?

5. A flu virus passes through the people living in a city at an exponential rate. If 10% of the population is infected after 10 days, how long will it take for 40% of the people to contract the flu?

6. Newton’s Law of Cooling states that the rate of change of the temperature of a warm object in a cool environment is proportional not to the temperature of the object, but to the *difference* in the temperature between the object and the environment.

- (a) Let y be the temperature of the object, y_0 be its initial temperature, T be the temperature of the environment, and k be the constant of proportionality. Identify a differential equation to model Newton’s Law of Cooling and find its solution.
(b) Foul play befalls a math professor who failed one too many students. The police discover his body (which has cooled from 37°C to 25°C) half an hour after his demise, on a MUN parking lot where the temperature is -8°C . The medical examiner arrives on the scene 15 minutes after the police. Use Newton’s Law of Cooling to find the temperature of the math prof’s body at this time.