

MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 3.1

Math 1001 Worksheet

WINTER 2024

For practice only. Not to be submitted.

1. Evaluate each of the following integrals using the method of partial fractions.

(a) $\int \frac{3x - 2}{x^2 - x} dx$

(b) $\int \frac{2 - x}{x^2 + 7x + 10} dx$

(c) $\int \frac{6x^3 + x^2 + 25x + 4}{x^4 + 7x^2 + 12} dx$

(d) $\int \frac{-2x^3 + 12x^2 + 162x}{x^4 - 81} dx$

2. An alternative way to find the constants in the partial fraction decomposition starts after multiplying the form of the decomposition by the original denominator. Instead of substituting values of x , the expression can be expanded and common terms collected. The coefficient of each power of x must then match the corresponding coefficients in the numerator of the original proper rational function. This approach can be particularly useful when the original denominator has a number of repeated factors. Consider, then, the function

$$f(x) = \frac{7 + 5x - 2x^2}{(x - 3)^3}.$$

- (a) The partial fraction decomposition has the form

$$f(x) = \frac{A}{x - 3} + \frac{B}{(x - 3)^2} + \frac{C}{(x - 3)^3}.$$

Show that the constants must obey the equation

$$(9A - 3B + C) + (B - 6A)x + Ax^2 = 7 + 5x - 2x^2.$$

- (b) Determine the value of the constants A , B and C .

- (c) Evaluate $\int f(x) dx$.