MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 7 **MATHEMATICS 1001** WINTER 2025

SOLUTIONS

[4] 1. (a) Observe that

$$x^2 - 3x - 10 = (x-5)(x+2),$$

so the denominator has two unique linear factors. Hence the form of the partial fraction decomposition is

$$\frac{11-5x}{x^2-3x-10} = \frac{A}{x-5} + \frac{B}{x+2}$$
$$11 - 5x = A(x+2) + B(x-5).$$

When $x = 5$ we have $-14 = 7A$ so $A = -2$. When $x = -2$ we have $21 = -7B$ so $B = -3$. Thus

$$\int \frac{11-5x}{x^2-3x-10}\,dx = \int \left[\frac{-2}{x-5} - \frac{3}{x+2}\right] dx$$
$$= -2\ln|x-5| - 3\ln|x+2| + C.$$

[6] (b) Observe that

$$4x^3 + 4x^2 + 16x + 16 = 4x^2(x+1) + 16(x+1) = (4x+4)(x^2+4),$$

so the denominator has one unique linear factor and one unique irreducible quadratic factor. Hence the form of the partial fraction decomposition is

$$\frac{13x^2-4x-12}{4x^3+4x^2+16x+16} = \frac{A}{4x+4} + \frac{Bx+C}{x^2+4}$$
$$13x^2 - 4x - 12 = A(x^2+4) + (Bx+C)(4x+4).$$

When $x = -1$ we have $5 = 5A$ so $A = 1$. When $x = 0$, we have $-12 = 4A + 4C$ so $4C = -12 - 4A = -16$ and $C = -4$. Finally when, say, $x = 1$, we have $-3 = 5A + 8B + 8C$ so $8B = -3 - 5A - 8C = 24$ and $B = 3$. Thus

$$\int \frac{13x^2-4x-12}{4x^3+4x^2+16x+16}\,dx = \int \left[\frac{\frac{1}{4}}{x+1} + \frac{3x-4}{x^2+4}\right] dx$$
$$= \int \left[\frac{\frac{1}{4}}{x+1} + \frac{3x}{x^2+4} - \frac{4}{x^2+4}\right] dx$$
$$= \frac{1}{4}\ln|x+1| - 2\arctan\left(\frac{x}{2}\right) + 3\int \frac{x}{x^2+4}\,dx.$$

For the remaining integral, we let $u = x^2 + 4$ so $\frac{1}{2}\,du = x\,dx$ and we have

$$\begin{aligned}\int \frac{13x^2 - 4x - 12}{4x^3 + 4x^2 + 16x + 16}\,dx &= \frac{1}{4}\ln|x+1| - 2\arctan\left(\frac{x}{2}\right) + \frac{3}{2}\int \frac{1}{u}\,du \\ &= \frac{1}{4}\ln|x+1| - 2\arctan\left(\frac{x}{2}\right) + \frac{3}{2}\ln|u| + C \\ &= \frac{1}{4}\ln|x+1| - 2\arctan\left(\frac{x}{2}\right) + \frac{3}{2}\ln(x^2+4) + C.\end{aligned}$$

[5] 2. (a) Since the power of cosine is odd, we set one factor aside and turn the remaining cosines into sines:

$$\begin{aligned}\int \sin^6(x)\cos^5(x)\,dx &= \int \sin^6(x)[\cos^2(x)]^2 \cdot \cos(x)\,dx \\ &= \int \sin^6(x)[1 - \sin^2(x)]^2 \cdot \cos(x)\,dx.\end{aligned}$$

Now we let $u = \sin(x)$ so $du = \cos(x)\,dx$. The integral becomes

$$\begin{aligned}\int \sin^6(x)\cos^5(x)\,dx &= \int u^6[1-u^2]^2\,du \\ &= \int u^6[1 - 2u^2 + u^4]\,du \\ &= \int [u^6 - 2u^8 + u^{10}]\,du \\ &= \frac{u^7}{7} - 2\cdot\frac{u^9}{9} + \frac{u^{11}}{11} + C \\ &= \frac{1}{7}\sin^7(x) - \frac{2}{9}\sin^9(x) + \frac{1}{11}\sin^{11}(x) + C.\end{aligned}$$

[5] (b) We write

$$\begin{aligned}
\int \cos^4(3x)\,dx &= \int [\cos^2(3x)]^2\,dx \\
&= \int \left[\frac{1+\cos(6x)}{2}\right]^2 dx \\
&= \frac{1}{4}\int [1 + 2\cos(6x) + \cos^2(6x)]\,dx \\
&= \frac{1}{4}\int \left[1 + 2\cos(6x) + \frac{1+\cos(12x)}{2}\right] dx \\
&= \frac{1}{4}\int \left[\frac{3}{2} + 2\cos(6x) + \frac{1}{2}\cos(12x)\right] dx \\
&= \frac{1}{4}\left[\frac{3}{2}x + \frac{1}{3}\sin(6x) + \frac{1}{24}\sin(12x)\right] + C \\
&= \frac{3}{8}x + \frac{1}{12}\sin(6x) + \frac{1}{96}\sin(12x) + C.
\end{aligned}$$