MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 3 | **MATHEMATICS 1001** | WINTER 2025

SOLUTIONS

[3] 1. (a) Let $u = \tan(t)$ so $du = \sec^2(t)\,dt$. The integral becomes

$$\begin{aligned}\int \frac{\sec^2(t)}{\sqrt{1-\tan^2(t)}}\,dt &= \int \frac{1}{\sqrt{1-u^2}}\,du \\ &= \arcsin(u) + C \\ &= \arcsin(\tan(t)) + C.\end{aligned}$$

[4] (b) We complete the square:

$$\begin{aligned}25x^2 - 20x + 53 &= 25\left[\left(x^2 - \frac{4}{5}x\right) + \frac{53}{25}\right] \\ &= 25\left[\left(x^2 - \frac{4}{5}x + \frac{4}{25}\right) + \frac{53}{25} - \frac{4}{25}\right] \\ &= 25\left[\left(x - \frac{2}{5}\right)^2 + \frac{49}{25}\right].\end{aligned}$$

At this point, we could further simplify this to obtain

$$25x^2 - 20x + 53 = (5x-2)^2 + 49.$$

Now we can write the given integral as

$$\int \frac{1}{25x^2 - 20x + 53}\,dx = \int \frac{1}{(5x-2)^2 + 49}\,dx.$$

Let $u = 5x - 2$ so $du = 5\,dx$ and $\frac{1}{5}\,du = dx$. Then

$$\begin{aligned}\int \frac{1}{25x^2 - 20x + 53}\,dx &= \frac{1}{5}\int \frac{1}{u^2 + 49}\,du \\ &= \frac{1}{5}\cdot\frac{1}{7}\arctan\left(\frac{u}{7}\right) + C \\ &= \frac{1}{35}\arctan\left(\frac{5x-2}{7}\right) + C.\end{aligned}$$

Alternatively, after completing the square we could write the integral as

$$\int \frac{1}{25x^2 - 20x + 53}\,dx = \int \frac{1}{25\left[\left(x - \frac{2}{5}\right)^2 + \frac{49}{25}\right]}\,dx = \frac{1}{25}\int \frac{1}{\left(x - \frac{2}{5}\right)^2 + \left(\frac{7}{5}\right)^2}\,dx.$$

Now we let $u = x - \frac{2}{5}$ so $du = dx$ and the integral becomes

$$\begin{aligned}
\int \frac{1}{25x^2 - 20x + 53}\, dx &= \frac{1}{25} \int \frac{1}{u + \left(\frac{7}{5}\right)^2}\, du \\
&= \frac{1}{25} \cdot \frac{1}{\frac{7}{5}} \arctan\left(\frac{u}{\frac{7}{5}}\right) + C \\
&= \frac{1}{35} \arctan\left(\frac{5}{7}\left(x - \frac{2}{5}\right)\right) + C \\
&= \boxed{\frac{1}{35} \arctan\left(\frac{5x - 2}{7}\right) + C.}
\end{aligned}$$

[5] 2. (a) Let $w = \cos(7x)$ so $dw = -7\sin(7x)\, dx$, and let $dv = \sin(x)\, dx$ so $v = -\cos(x)$. Now we have

$$\int \sin(x)\cos(7x)\, dx = -\cos(x)\cos(7x) - 7\int \cos(x)\sin(7x)\, dx.$$

We use integration by parts again, this time letting $w = \sin(7x)$ so $dw = 7\cos(7x)\, dx$, and $dv = \cos(x)\, dx$ so $v = \sin(x)$. Hence

$$\begin{aligned}
\int \sin(x)\cos(7x)\, dx &= -\cos(x)\cos(7x) - 7\left[\sin(x)\sin(7x) - 7\int \sin(x)\cos(7x)\, dx\right] \\
&= -\cos(x)\cos(7x) - 7\sin(x)\sin(7x) + 49\int \sin(x)\cos(7x)\, dx \\
-48\int \sin(x)\cos(7x)\, dx &= -\cos(x)\cos(7x) - 7\sin(x)\sin(7x) \\
\int \sin(x)\cos(7x)\, dx &\boxed{= \frac{1}{48}\cos(x)\cos(7x) + \frac{7}{48}\sin(x)\sin(7x) + C.}
\end{aligned}$$

Alternatively, we could let $w = \sin(x)$ so $dw = \cos(x)\, dx$, and let $dv = \cos(7x)\, dx$ so $v = \frac{1}{7}\sin(7x)$. Now we have

$$\int \sin(x)\cos(7x)\, dx = \frac{1}{7}\sin(x)\sin(7x) - \frac{1}{7}\int \cos(x)\sin(7x)\, dx.$$

We use integration by parts again, this time letting $w = \cos(x)$ so $dw = -\sin(x)\, dx$, and

$dv = \sin(7x)\,dx$ so $v = -\frac{1}{7}\cos(7x)$. Hence

$$\begin{aligned}\int \sin(x)\cos(7x)\,dx &= \frac{1}{7}\sin(x)\sin(7x) \\ &\qquad - \frac{1}{7}\left[-\frac{1}{7}\cos(x)\cos(7x) - \frac{1}{7}\int \sin(x)\cos(7x)\,dx\right] \\ &= \frac{1}{7}\sin(x)\sin(7x) + \frac{1}{49}\cos(x)\cos(7x) + \frac{1}{49}\int \sin(x)\cos(7x)\,dx \\ \frac{48}{49}\int \sin(x)\cos(7x)\,dx &= \frac{1}{7}\sin(x)\sin(7x) + \frac{1}{49}\cos(x)\cos(7x) \\ \int \sin(x)\cos(7x)\,dx &= \frac{7}{48}\sin(x)\sin(7x) + \frac{1}{48}\cos(x)\cos(7x) + C.\end{aligned}$$

[4] (b) Let $w = \arccos(x)$ so $dw = \frac{-1}{\sqrt{1-x^2}}\,dx$, and let $dv = dx$ so $v = x$. We obtain

$$\int \arccos(x)\,dx = x\arccos(x) + \int \frac{x}{\sqrt{1-x^2}}\,dx.$$

Now let $u = 1 - x^2$ so $du = -2x\,dx$ and $-\frac{1}{2}\,dx = x\,dx$. The integral becomes

$$\begin{aligned}\int \arccos(x)\,dx &= x\arccos(x) - \frac{1}{2}\int \frac{1}{\sqrt{u}}\,du \\ &= x\arccos(x) - \frac{1}{2}\int u^{-\frac{1}{2}}\,du \\ &= x\arccos(x) - \frac{1}{2}\cdot\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C \\ &= x\arccos(x) - \sqrt{1-x^2} + C.\end{aligned}$$

[4] (c) Let $u = x^3 + 1$ so $du = 3x^2\,dx$ and $\frac{1}{3}\,du = x^2\,dx$. Furthermore, $x^3 = u - 1$. The integral becomes

$$\begin{aligned}\int x^5\sin(x^3+1)\,dx &= \int x^3\sin(x^3+1)\cdot x^2\,dx \\ &= \frac{1}{3}\int (u-1)\sin(u)\,du.\end{aligned}$$

Now let $w = u - 1$ so $dw = du$, and let $dv = \sin(u)\,du$ so $v = -\cos(u)$. This yields

$$\begin{aligned}\int x^5\sin(x^3+1)\,dx &= \frac{1}{3}\left[-(u-1)\cos(u) + \int \cos(u)\,du\right] \\ &= -\frac{1}{3}(u-1)\cos(u) + \frac{1}{3}\sin(u) + C \\ &= -\frac{1}{3}x^3\cos(x^3-1) + \frac{1}{3}\sin(x^3-1) + C.\end{aligned}$$