MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

Assignment 3 | **MATHEMATICS 1001** | Winter 2024

SOLUTIONS

[5] 1. (a) Complete the square:

$$\begin{aligned} 16-6t-t^2 &= -[t^2+6t-16] \\ &= -[(t^2+6t+9)-16-9] \\ &= -[(t+3)^2-25] \\ &= 25-(t+3)^2. \end{aligned}$$

Thus

$$\int \frac{1}{\sqrt{16-6t-t^2}}\,dt = \int \frac{1}{\sqrt{25-(t+3)^2}}\,dt.$$

Let $u = t+3$ so $du = dt$. Then

$$\begin{aligned} \int \frac{1}{\sqrt{16-6t-t^2}}\,dt &= \int \frac{1}{\sqrt{25-u^2}}\,du \\ &= \arcsin\left(\frac{u}{5}\right)+C \\ &= \arcsin\left(\frac{t+3}{5}\right)+C. \end{aligned}$$

[3] (b) Let $u = \sin(x)$ so $du = \cos(x)\,dx$. The integral becomes

$$\begin{aligned} \int \frac{\cos(x)}{\sin^2(x)+6}\,dx &= \int \frac{1}{u^2+6}\,du \\ &= \frac{1}{\sqrt{6}}\arctan\left(\frac{u}{\sqrt{6}}\right)+C \\ &= \frac{\sqrt{6}}{6}\arctan\left(\frac{\sqrt{6}u}{6}\right)+C \\ &= \frac{\sqrt{6}}{6}\arctan\left(\frac{\sqrt{6}\sin(x)}{6}\right)+C. \end{aligned}$$

[4] 2. (a) Let $w = x^2$ so $dw = 2x\,dx$, and let $dv = \sin(2x)\,dx$ so $v = -\frac{1}{2}\cos(2x)$. Then

$$\int x^2\sin(2x)\,dx = -\frac{1}{2}x^2\cos(2x) + \int x\cos(2x)\,dx.$$

We use integration by parts a second time, with $w = x$ so $dw = dx$, and $dv = \cos(2x)\,dx$ so $v = \frac{1}{2}\sin(2x)$. Now

$$\int x\cos(2x)\,dx = \frac{1}{2}x\sin(2x) - \frac{1}{2}\int \sin(2x)\,dx$$
$$= \frac{1}{2}x\sin(2x) + \frac{1}{4}\cos(2x) + C.$$

This means that

$$\int x^2\sin(2x)\,dx = -\frac{1}{2}x^2\cos(2x) + \frac{1}{2}x\sin(2x) + \frac{1}{4}\cos(2x) + C.$$

[4] (b) First we use u-substitution, with $u = \frac{1}{x}$ so $du = -x^{-2}\,dx$. The integral becomes

$$\int x^{-3}e^{\frac{1}{x}}\,dx = -\int x^{-1}e^u\,du = -\int ue^u\,du.$$

Now we use integration by parts, letting $w = u$ so $dw = du$, and $dv = e^u\,du$ so $v = e^u$. Then

$$\int x^{-3}e^{\frac{1}{x}}\,dx = -\left[ue^u - \int e^u\,du\right]$$
$$= -ue^u + e^u + C$$
$$= -\frac{e^{\frac{1}{x}}}{x} + e^{\frac{1}{x}} + C.$$

[4] (c) Let $w = \operatorname{arcsec}(x)$ so $dw = \dfrac{1}{x\sqrt{x^2-1}}\,dx$. Let $dv = x\,dx$ so $v = \frac{1}{2}x^2$. Then we have

$$\int x\operatorname{arcsec}(x)\,dx = \frac{1}{2}x^2\operatorname{arcsec}(x) - \frac{1}{2}\int x^2\cdot\frac{1}{x\sqrt{x^2-1}}\,dx$$
$$= \frac{1}{2}x^2\operatorname{arcsec}(x) - \frac{1}{2}\int \frac{x}{\sqrt{x^2-1}}\,dx.$$

Now let $u = x^2 - 1$ so $du = 2x\,dx$ and $\frac{1}{2}\,du = x\,dx$. The integral becomes

$$\int x\operatorname{arcsec}(x)\,dx = \frac{1}{2}x^2\operatorname{arcsec}(x) - \frac{1}{4}\int u^{-\frac{1}{2}}\,du$$
$$= \frac{1}{2}x^2\operatorname{arcsec}(x) - \frac{1}{4}\cdot 2\sqrt{u} + C$$
$$= \frac{1}{2}x^2\operatorname{arcsec}(x) - \frac{1}{2}\sqrt{x^2-1} + C.$$