MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

Assignment 2	**MATHEMATICS 1001**	Winter 2025

SOLUTIONS

[3] 1. (a) Let $u = e^x$ so $du = e^x\,dx$. The integral becomes

$$\int \sec(u)\,du = \ln|\sec(u) + \tan(u)| + C$$
$$= \ln|\sec(e^x) + \tan(e^x)| + C.$$

[6] (b) Let $u = 2x^2 + 1$ so $du = 4x\,dx$ and $\frac{1}{4}\,du = x\,dx$. Then

$$\int \frac{x^3}{\sqrt{2x^2+1}}\,dx = \int \frac{x^2}{\sqrt{2x^2+1}} \cdot x\,dx = \int \frac{x^2}{\sqrt{u}} \cdot \frac{1}{4}\,du.$$

Now we need to solve for x^2 in terms of u. We have

$$2x^2 = u - 1 \quad \Longrightarrow \quad x^2 = \frac{1}{2}u - \frac{1}{2}.$$

Hence the integral becomes

$$\int \frac{\frac{1}{2}u - \frac{1}{2}}{\sqrt{u}} \cdot \frac{1}{4}\,du = \frac{1}{8}\int \frac{u-1}{\sqrt{u}}\,du$$
$$= \frac{1}{8}\int \left(u^{\frac{1}{2}} - u^{-\frac{1}{2}}\right)\,du$$
$$= \frac{1}{8}\left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}} - \frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right] + C$$
$$= \frac{1}{12}u^{\frac{3}{2}} - \frac{1}{4}\sqrt{u} + C$$
$$= \frac{1}{12}(2x^2+1)^{\frac{3}{2}} - \frac{1}{4}\sqrt{2x^2+1} + C.$$

[4] (c) We let $u = 4x^3 - 2x + 3$ so $du = (12x^2 - 2)\,dx = -2(1-6x^2)\,dx$ and $-\frac{1}{2}\,du = (1-6x^2)\,dx$. The integral becomes

$$\int \frac{1-6x^2}{4x^3-2x+3}\,dx = -\frac{1}{2}\int \frac{1}{u}\,du$$
$$= -\frac{1}{2}\ln|u| + C$$
$$= -\frac{1}{2}\ln|4x^3-2x+3| + C.$$

[7] (d) We can rewrite the integrand using long division. Observe that

$$\begin{array}{r} x^2+3x-4 \\ x^2+2\,\overline{\smash{)}\,x^4+3x^3-2x^2+x-8} \\ \underline{x^4\qquad\quad+2x^2}\qquad\qquad \\ 3x^3-4x^2+x-8 \\ \underline{3x^3\qquad\quad+6x}\quad\;\; \\ -4x^2-5x-8 \\ \underline{-4x^2\qquad\;\;-8} \\ -5x\quad\;\; \end{array}$$

Thus we can write

$$\begin{aligned} \int \frac{x^4+3x^3-2x^2+x-8}{x^2+2}\,dx &= \int \left(x^2+3x-4-\frac{5x}{x^2+2}\right)\,dx \\ &= \frac{1}{3}x^3+\frac{3}{2}x^2-4x-5\int \frac{x}{x^2+2}\,dx. \end{aligned}$$

To evaluate the remaining integral, we set $u = x^2+2$ so $du = 2x\,dx$ and $\frac{1}{2}\,du = x\,dx$. Thus

$$\begin{aligned} \int \frac{x^4+3x^3-2x^2+x-8}{x^2+2}\,dx &= \frac{1}{3}x^3+\frac{3}{2}x^2-4x-\frac{5}{2}\int \frac{1}{u}\,du \\ &= \frac{1}{3}x^3+\frac{3}{2}x^2-4x-\frac{5}{2}\ln|u|+C \\ &= \frac{1}{3}x^3+\frac{3}{2}x^2-4x-\frac{5}{2}\ln|x^2+2|+C \\ &= \frac{1}{3}x^3+\frac{3}{2}x^2-4x-\frac{5}{2}\ln(x^2+2)+C. \end{aligned}$$