

MEMORIAL UNIVERSITY OF NEWFOUNDLAND
DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 2

MATHEMATICS 1001

WINTER 2024

SOLUTIONS

- [4] 1. (a) Let $u = \sqrt{3t}$ so $du = \frac{\sqrt{3}}{2\sqrt{t}} dt$ and $\frac{2}{3} du = \frac{1}{\sqrt{3t}} dt$. The integral becomes

$$\begin{aligned}\int \frac{6 \tan(\sqrt{3t})}{\sqrt{3t}} dt &= \frac{2}{3} \cdot 6 \int \tan(u) du \\ &= 4[-\ln|\cos(u)|] + C \\ &= -4 \ln|\cos(\sqrt{3t})| + C.\end{aligned}$$

- [6] (b) Let $u = 3 - x^2$ so $du = -2x dx$ and $-\frac{1}{2} du = x dx$. Furthermore, $x^2 = 3 - u$ so $x^4 = (3 - u)^2$. The integral becomes

$$\begin{aligned}\int x^5(3 - x^2)^7 dx &= -\frac{1}{2} \int x^4 u^7 du \\ &= -\frac{1}{2} \int (3 - u)^2 u^7 du \\ &= -\frac{1}{2} \int (9u^7 - 6u^8 + u^9) du \\ &= -\frac{1}{2} \left[\frac{9}{8} u^8 - \frac{2}{3} u^9 + \frac{1}{10} u^{10} \right] + C \\ &= -\frac{9}{16} (3 - x^2)^8 + \frac{1}{3} (3 - x^2)^9 - \frac{1}{20} (3 - x^2)^{10} + C.\end{aligned}$$

- [4] (c) We use u -substitution with $u = 5x^3 - 6x^2$ so $du = (15x^2 - 12x) dx$ and $\frac{1}{3} du = (5x^2 - 4x) dx$. Now we can write

$$\begin{aligned}\int \frac{10x^2 - 8x}{5x^3 - 6x^2} dx &= 2 \int \frac{5x^2 - 4x}{5x^3 - 6x^2} dx \\ &= \frac{2}{3} \int \frac{1}{u} du \\ &= \frac{2}{3} \ln|u| + C \\ &= \frac{2}{3} \ln|5x^3 - 6x^2| + C.\end{aligned}$$

[6] (d) We begin by performing long division:

$$\begin{array}{r} 3x \\ x^3 + 2 \overline{) 3x^4 - x^2 + 6x} \\ \underline{3x^4} \\ -x^2 \end{array}$$

Thus we can write

$$\int \frac{3x^4 - x^2 + 6x}{x^3 + 2} dx = \int \left(3x - \frac{x^2}{x^3 + 2} \right) dx = \frac{3}{2}x^2 - \int \frac{x^2}{x^3 + 2} dx.$$

For the remaining integral, let $u = x^3 + 2$ so $du = 3x^2 dx$ and $\frac{1}{3} du = x^2 dx$. Then

$$\begin{aligned} \int \frac{3x^4 - x^2 + 6x}{x^3 + 2} dx &= \frac{3}{2}x^2 - \frac{1}{3} \int \frac{1}{u} du \\ &= \frac{3}{2}x^2 - \frac{1}{3} \ln|u| + C \\ &= \frac{3}{2}x^2 - \frac{1}{3} \ln|x^3 + 2| + C. \end{aligned}$$