MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 1 **MATHEMATICS 1001** WINTER 2024

SOLUTIONS

[3] 1. (a) We can multiply out to get

$$\begin{aligned}\int x^{\frac{3}{2}}\left(x^2 - x^{-2}\right)\,dx &= \int \left(x^{\frac{7}{2}} - x^{-\frac{1}{2}}\right)\,dx\\ &= \frac{2}{9}x^{\frac{9}{2}} - 2x^{\frac{1}{2}} + C\\ &= \frac{2}{9}x^{\frac{9}{2}} - 2\sqrt{x} + C.\end{aligned}$$

[2] (b) First recall the trigonometric identity

$$\csc^2(x) - \cot^2(x) = 1.$$

Then the integral can be rewritten

$$\begin{aligned}\int [3\csc^2(x) - 3\cot^2(x)]\,dx &= 3\int [\csc^2(x) - \cot^2(x)]\,dx\\ &= 3\int dx\\ &= 3x + C.\end{aligned}$$

[3] (c) Expanding the integrand yields

$$\begin{aligned}\int \sec(u)[7\cos(u) + 2\tan(u)]\,du &= \int [7 + 2\sec(u)\tan(u)]\,du\\ &= 7u + 2\sec(u) + C.\end{aligned}$$

[3] (d) Since

$$\int x^{10}\,dx = \frac{x^{11}}{11} + C,$$

we have

$$\begin{aligned}\int \frac{(9+2x)^{10}}{6}\,dx &= \frac{1}{6}\int (9+2x)^{10}\,dx\\ &= \frac{1}{6}\left[\frac{1}{2}\cdot\frac{(9+2x)^{11}}{11}\right] + C\\ &= \frac{1}{132}(9+2x)^{11} + C.\end{aligned}$$

[3] (e) Since

$$\int x^{-10}\,dx = \frac{x^{-9}}{-9} + C,$$

we can write

$$\int \frac{6}{(9+2x)^{10}}\,dx = 6\int (9+2x)^{-10}\,dx$$
$$= 6\left[\frac{1}{2}\cdot\frac{(9+2x)^{-9}}{-9}\right] + C$$
$$= -\frac{1}{3(9+2x)^9} + C.$$

[3] (f) Since

$$\int \cos(t)\,dt = \sinh(t) + C \quad \text{and} \quad \int \csc^2(t)\,dt = -\cot(t) + C,$$

we have

$$\int \left[\cosh(1-3t) - \csc^2\left(\frac{t}{5}\right)\right]\,dt = \frac{1}{-3}\sinh(1-3t) - \frac{1}{\frac{1}{5}}\left[-\cot\left(\frac{t}{5}\right)\right] + C$$
$$= -\frac{1}{3}\sinh(1-3t) + 5\cot\left(\frac{t}{5}\right) + C.$$

[3] (g) First we rewrite the integrand as

$$\int \frac{e^{7x}+6e^x}{2e^{4x}}\,dx = \int \left(\frac{1}{2}e^{3x} + 3e^{-3x}\right)\,dx.$$

Now we can use the fact that

$$\int e^x\,dx = e^x + C$$

and conclude that

$$\int \frac{e^{7x}+6e^x}{2e^{4x}}\,dx = \frac{1}{2}\cdot\frac{1}{3}e^{3x} + 3\cdot\frac{1}{-3}e^{-3x} + C$$
$$= \frac{1}{6}e^{3x} - \frac{1}{e^{3x}} + C.$$