Section 1.1

Common integrals:

① $\int 0 \, dx = C$

② $\int 1 \, dx = \int dx = x + C$

③ $\int x^r \, dx = \frac{x^{r+1}}{r+1} + C \quad , \quad r \neq -1$

④ $\int x^{-1} \, dx = \int \frac{1}{x} \, dx = \ln|x| + C$

⑤ $\int e^x \, dx = e^x + C$

Recall that $[b^x]' = b^x \ln(b)$ for $b > 0$, $b \neq 1$

$$\left[\frac{b^x}{\ln(b)}\right]' = \frac{b^x \ln(b)}{\ln(b)} = b^x$$

⑥ $\int b^x \, dx = \frac{b^x}{\ln(b)} + C$

⑦ $\int \cos(x) \, dx = \sin(x) + C$

⑧ $\int \sin(x) \, dx = -\cos(x) + C$

⑨ $\int \sec(x) \tan(x) \, dx = \sec(x) + C$

⑩ $\int \csc(x) \cot(x) \, dx = -\csc(x) + C$

⑪ $\int \sec^2(x) \, dx = \tan(x) + C$

⑫ $\int \csc^2(x) \, dx = -\cot(x) + C$

(13) $\int \cosh(x)\,dx = \sinh(x) + C$

(14) $\int \sinh(x)\,dx = \cosh(x) + C$

Theorem: Basic Properties of Indefinite Integration

If $f(x)$ and $g(x)$ are integrable functions then

(1) $\int k f(x)\,dx = k \int f(x)\,dx$ for any constant k

(2) $\int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$

(3) $\int [f(x) - g(x)]\,dx = \int f(x)\,dx - \int g(x)\,dx$

eg $\int (x^5 - 4x^2 + 6)\,dx$

$= \int x^5\,dx - 4\int x^2\,dx + 6\int dx$

$= \left[\frac{x^6}{6} + C_1\right] - 4\left[\frac{x^3}{3} + C_2\right] + 6\left[x + C_3\right]$

$= \frac{1}{6}x^6 - \frac{4}{3}x^3 + 6x + C_1 - 4C_2 + 6C_3$

$\boxed{= \frac{1}{6}x^6 - \frac{4}{3}x^3 + 6x + C}$

eg $\int \left[\pi \sin(x) - \frac{1}{2}\cos(x)\right] dx$

$= \pi \int \sin(x)\,dx - \frac{1}{2}\int \cos(x)\,dx$

$= \pi\left[-\cos(x)\right] - \frac{1}{2}\left[\sin(x)\right] + C$

$\boxed{= -\pi\cos(x) - \frac{1}{2}\sin(x) + C}$

eg $\int \left(\frac{x^2}{3} - \frac{4}{5x^3} + \frac{3}{x} \right) dx$

$$= \frac{1}{3} \int x^2 \, dx - \frac{4}{5} \int x^{-3} \, dx + 3 \int \frac{1}{x} \, dx$$

$$= \frac{1}{3} \left[\frac{x^3}{3} \right] - \frac{4}{5} \left[\frac{x^{-2}}{-2} \right] + 3 \cdot \ln|x| + C$$

$$\boxed{= \frac{1}{9} x^3 + \frac{2}{5} x^{-2} + 3 \ln|x| + C}$$

Unfortunately, there is no general method for integrating products, quotients or composite functions.

However some integrals involving these operations can be evaluated by first rewriting the integrand so that it is just a sum, difference or constant multiple of the common integrals.

eg $\int (\sqrt{x} - 1)^2 \, dx$

$$= \int (\sqrt{x} - 1)(\sqrt{x} - 1) \, dx$$

$$= \int (x - 2\sqrt{x} + 1) \, dx$$

$$= \int x \, dx - 2 \int x^{1/2} \, dx + \int dx$$

$$= \frac{1}{2} x^2 - 2 \left[\frac{x^{3/2}}{3/2} \right] + x + C$$

$$\boxed{= \frac{1}{2} x^2 - \frac{4}{3} x^{3/2} + x + C}$$

eg $\displaystyle\int \frac{\sin(\theta)}{\cos^2(\theta)}\, d\theta$

$$= \int \frac{1}{\cos(\theta)} \cdot \frac{\sin(\theta)}{\cos(\theta)}\, d\theta$$

$$= \int \sec(\theta)\tan(\theta)\, d\theta \; \boxed{= \sec(\theta) + C}$$

Now consider the linear composition of a simple function and explore how this affects its integral. In general, this has the form

$$\int f(mx+b)\, dx$$

where $\int f(x)\,dx = F(x) + C$ is known, and m and b are constants where $m \neq 0$.

eg $\displaystyle\int (2x-3)^{60}\, dx$

Compare: $\displaystyle\int x^{60}\, dx = \frac{x^{61}}{61} + C$

Is this as simple as

$$\int (2x-3)^{60}\, dx = \frac{(2x-3)^{61}}{61} + C \; ?$$