Section 1.1: Indefinite Integration

Def'n: An antiderivative of the function $f(x)$ is a function $F(x)$ such that $F'(x) = f(x)$ for all x at which $f(x)$ is defined.

The process of finding an antiderivative is called antidifferentiation.

eg We know that $[x^2]' = 2x$ so x^2 is an antiderivative of $2x$.

But $[x^2+1]' = 2x$ as well, so x^2+1 is also an antiderivative of $2x$.

Likewise, $[x^2 - \sqrt{3}]' = 2x$ so $x^2 - \sqrt{3}$ is another antiderivative of $2x$.

In fact, $x^2 + C$ for any real constant C is an antiderivative of $2x$. We call this "most general" antiderivative the indefinite integral of $2x$.

Def'n: If $f(x)$ is a function with antiderivative $F(x)$ then the indefinite integral of $f(x)$ is $F(x) + C$ where C is an arbitrary real constant known as the constant of integration. If $f(x)$ possesses an indefinite integral then it is integrable. The process of finding the indefinite integral is indefinite integration.

eg We know that $[\sin(x)]' = \cos(x)$ so the indefinite integral of $\cos(x)$ is $\sin(x) + C$.

We denote the indefinite integral of $f(x)$ as

$$\int f(x)\,dx$$

where $\int$ is the integral symbol and dx is called a differential. The function $f(x)$ being integrated is the integrand.

eg $\int 2x\,dx = x^2 + C$

$\int \cos(x)\,dx = \sin(x) + C$

The general process for indefinite integration is as follows:

1. Rewrite the integrand in a more convenient form (if necessary)
2. Identify the indefinite integral based on our knowledge of derivatives
3. Perform any obvious simplifications (if possible)

eg $\int \frac{5x^{9/2}}{\sqrt{x}}\,dx$

We can rewrite the integrand:

$$\frac{5x^{9/2}}{\sqrt{x}} = \frac{5x^{9/2}}{x^{1/2}} = 5x^4$$

So, from the Power Rule for derivatives,

$$\int 5x^4\,dx \boxed{= x^5 + C}$$

We can use our results for derivatives to establish several <u>common integrals</u>:

1. $\int 0\,dx = C$
2. $\int 1\,dx = \int dx = x + C$

The Power Rule for derivatives states that

$$[x^r]' = rx^{r-1}$$

so $\int rx^{r-1}\,dx = x^r + C$.

Likewise, $[x^{r+1}]' = (r+1)\,x^r$

$$\left[\frac{x^{r+1}}{r+1}\right]' = \frac{(r+1)\,x^r}{r+1} = x^r$$

So now we have:

(3) $$\int x^r\,dx = \frac{x^{r+1}}{r+1} + C \quad \text{for } r \neq -1$$

which is the <u>Power Rule for integrals</u>

eg $\int x^7\,dx \boxed{= \frac{x^8}{8} + C = \frac{1}{8}x^8 + C}$

eg $$\int \sqrt{\frac{1}{x^3}}\,dx = \int \frac{1}{x^{3/2}}\,dx = \int x^{-3/2}\,dx$$

$$= \frac{x^{-1/2}}{-1/2} + C$$

$$\boxed{\begin{aligned} &= -2x^{-1/2} + C \\ &= \frac{-2}{\sqrt{x}} + C \end{aligned}}$$

What about $\int x^{-1}\,dx = \int \frac{1}{x}\,dx$?

We know that $[\ln(x)]' = \frac{1}{x}$ but $\ln(x)$ is defined only for $x > 0$ while $\frac{1}{x}$ is defined for all $x \neq 0$.

But observe that $[\ln(-x)]' = \frac{1}{-x} \cdot (-1) = \frac{1}{x}$.

Thus $\int \frac{1}{x}\,dx = \ln(x) + C$ for $x > 0$ and

$\int \frac{1}{x}\,dx = \ln(-x) + C$ for $x < 0$ or, put together:

$$④ \quad \int \frac{1}{x}\,dx = \ln|x| + C$$