

MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

AREA UNDER A CURVE

Math 1001 Worksheet

FALL 2019

For practice only. Not to be submitted.

1. Write each of the following sums using sigma notation (with $i = 1$ as the lower bound of summation).

(a) $\frac{2}{5} + \frac{4}{10} + \frac{8}{15} + \frac{16}{20} + \cdots + \frac{2^n}{5i}$

(b) $y^2 + 8y^2 + 27y^2 + 64y^2 + \cdots + n^3y^2$

2. Use the summation formulas to evaluate each of the following sums.

(a) $\sum_{i=1}^n (4i + 3)$

(b) $\sum_{i=1}^n (i^3 - 6i)$

(c) $\sum_{i=1}^n (i + 4)^2$

3. Consider the region R which lies between the graph of $f(x) = 2 - x$ and the x -axis, on the interval $[-1, 1]$.

(a) Determine the width Δx of each subinterval in a regular partition of $[-1, 1]$.

(b) Give an expression for the right endpoint x_i of the i th subinterval. Use this to find formulas for the point m_i and M_i at which $f(x)$ attains its minimum and maximum values on the i th subinterval.

(c) Determine $f(m_i)$ and $f(M_i)$.

(d) Find formulas for the lower sum $s(n)$ and the upper sum $S(n)$.

(e) Estimate the area A of the region R by finding the lower and upper sums with $n = 5$ subintervals. Give your answers to one decimal place.

(f) Estimate the area A of the region R by finding the lower and upper sums with $n = 500$ subintervals. Give your answers to two decimal places.

(g) Find the true value of A by taking the limit of the formulas you derived in part (d) as $n \rightarrow \infty$, and showing that these limits are equal.

4. Use the limit of a Riemann sum to determine the area A of the region which lies below the curve $f(x) = 3 + 3x - x^2$, above the x -axis, to the left of $x = -1$ and to the right of $x = 3$.