MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

GROWTH AND DECAY | **Math 1001 Worksheet** | FALL 2019

SOLUTIONS

1. (a) Let $y(t)$ be the amount of Einsteinium-254 left, in milligrams, after t days, so $y(t) = Ce^{kt}$. Then we first know that $y(0) = C = 3$. Since half the sample is left after 270 days, this implies

$$y(270) = 3e^{270k} = 1.5 \quad \Longrightarrow \quad k = -\frac{1}{270}\ln(2) \quad \Longrightarrow \quad y(t) = 3e^{-\frac{t}{270}\ln(2)}.$$

Thus, after 30 days, there will be

$$y(30) = 3e^{-\frac{30}{270}\ln(2)} \approx 2.38.$$

There will be about 2.38 mg left after 30 days.

(b) We want to know when $y(t) = 0.5$, so from part (a) we set

$$3e^{-\frac{t}{270}\ln(2)} = 0.5 \quad \Longrightarrow \quad t = \frac{270\ln(6)}{\ln(2)} \approx 698.$$

So it takes about 698 days for the sample to be reduced to 0.5 mg.

2. Let $y(t)$ be the number of parakeets on the island after t years, so $y(t) = Ce^{kt}$. Then we know that

$$y(2) = Ce^{2k} = 50 \quad \text{and} \quad y(5) = Ce^{5k} = 150.$$

Dividing the second by the first gives

$$\frac{Ce^{5k}}{Ce^{2k}} = \frac{150}{50} \quad \Longrightarrow \quad e^{3k} = 3 \quad \Longrightarrow \quad k = \frac{1}{3}\ln(3).$$

Hence using $y(2) = 50$ we have

$$50 = Ce^{\frac{2}{3}\ln(3)} \quad \Longrightarrow \quad C = 50e^{-\frac{2}{3}\ln(3)} \approx 24.$$

So there were about 24 parakeets originally on the island.

3. Let $y(t)$ be the number of healthy individuals after t days, so $y(t) = Ce^{kt}$. If there are C people in the city, after 10 days 10% (that is, $\frac{1}{10}$) of them have contracted the flu, so only $\frac{9}{10}C$ people are healthy. Hence

$$y(10) = Ce^{10k} = \frac{9}{10}C \quad \Longrightarrow \quad e^{10k} = \frac{9}{10} \quad \Longrightarrow \quad k = \frac{1}{10}\ln\left(\frac{9}{10}\right).$$

We want to know when 40% of the population is infected, so 60% or $\frac{3}{5}C$ people remain healthy, implying

$$y(t) = Ce^{\frac{t}{10}\ln\left(\frac{9}{10}\right)} = \frac{3}{5}C \quad \Longrightarrow \quad e^{\frac{t}{10}\ln\left(\frac{9}{10}\right)} = \frac{3}{5} \quad \Longrightarrow \quad t = 10\frac{\ln\left(\frac{3}{5}\right)}{\ln\left(\frac{9}{10}\right)} \approx 48.5.$$

So it will take about 48.5 days for 40% of the people to contract the flu.

4. Let $y(t)$ be the temperature of the body after t minutes, so by Newton's Law of Cooling, $y(t) = Ce^{kt} + T$ where $T = -8$ is the temperature of the surroundings. Then

$$y(0) = C + T = C - 8 = 37 \quad \Longrightarrow \quad C = 45.$$

Furthermore,

$$y(30) = 45e^{30k} - 8 = 25 \quad \Longrightarrow \quad k = \frac{1}{30}\ln\left(\frac{33}{45}\right).$$

Hence

$$y(45) = 45e^{\frac{45}{30}\ln\left(\frac{33}{45}\right)} - 8 \approx 20.3.$$

So the temperature of the body when the medical examiner arrives is about 20.3°C.