MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 5 | **MATHEMATICS 1001** | FALL 2025

SOLUTIONS

[4] 1. We write

$$\int_{6x}^{x^6} \sec(t^2)\,dt = \int_{6x}^{0} \sec(t^2)\,dt + \int_{0}^{x^6} \sec(t^2)\,dt = -\int_{0}^{6x} \sec(t^2)\,dt + \int_{0}^{x^6} \sec(t^2)\,dt.$$

Then

$$f'(x) = -\sec((6x)^2)\cdot[6x]' + \sec((x^6)^2)\cdot[x^6]' = -6\sec(36x^2) + 6x^5\sec(x^{12}).$$

[4] 2. (a) Let $u = \pi\sqrt{x}$ so $\frac{2}{\pi}\,du = \frac{1}{\sqrt{x}}\,dx$. When $x = \frac{1}{4}$, $u = \frac{\pi}{2}$. When $x = 4$, $u = 2\pi$. The integral becomes

$$\begin{aligned}\int_{\frac{1}{4}}^{4} \frac{\cos(\pi\sqrt{x})}{\sqrt{x}}\,dx &= \frac{2}{\pi}\int_{\frac{\pi}{2}}^{2\pi} \cos(u)\,du \\ &= \frac{2}{\pi}\Big[\sin(u)\Big]_{\frac{\pi}{2}}^{2\pi} \\ &= \frac{2}{\pi}\left[\sin(2\pi) - \sin\left(\frac{\pi}{2}\right)\right] \\ &= -\frac{2}{\pi}.\end{aligned}$$

[4] (b) Observe that

$$|5t-2| = \begin{cases} 5t-2, & \text{for } t \geq \frac{2}{5} \\ -(5t-2), & \text{for } t < \frac{2}{5}. \end{cases}$$

Thus we can write

$$\begin{aligned}&\int_{-1}^{3} |5t-2|\,dt \\ &= \int_{-1}^{\frac{2}{5}} |5t-2|\,dt + \int_{\frac{2}{5}}^{3} |5t-2|\,dt \\ &= -\int_{-1}^{\frac{2}{5}} (5t-2)\,dt + \int_{\frac{2}{5}}^{3} (5t-2)\,dt \\ &= -\left[\frac{5}{2}t^2 - 2t\right]_{-1}^{\frac{2}{5}} + \left[\frac{5}{2}t^2 - 2t\right]_{\frac{2}{5}}^{3} \\ &= -\left[\left(\frac{5}{2}\cdot\frac{4}{25} - 2\cdot\frac{2}{5}\right) - \left(\frac{5}{2} - 2(-1)\right)\right] + \left[\left(\frac{5}{2}\cdot 9 - 2(3)\right) - \left(\frac{5}{2}\cdot\frac{4}{25} - 2\cdot\frac{2}{5}\right)\right] \\ &= \frac{109}{5}.\end{aligned}$$

[4] (c) Let $u = \sec(x)$ so $du = \sec(x)\tan(x)\,dx$. When $x = 0$, $u = \sec(0) = \frac{1}{\cos(0)} = 1$. When $x = \frac{\pi}{3}$, $u = \sec(\frac{\pi}{3}) = \frac{1}{\cos(\frac{\pi}{3})} = 2$. The integral becomes

$$\int_0^{\frac{\pi}{3}} \sec^2(x)\tan(x)e^{\sec(x)}\,dx = \int_1^2 ue^u\,du.$$

Now we use integration by parts, letting $w = u$ so $dw = du$, and $dv = e^u\,du$ so $v = e^u$. Thus we have

$$\begin{aligned}\int_0^{\frac{\pi}{3}} \sec^2(x)\tan(x)e^{\sec(x)}\,dx &= \Big[ue^u\Big]_1^2 - \int_1^2 e^u\,du \\ &= \Big[ue^u - e^u\Big]_1^2 \\ &= [2e^2 - e^2] - [e - e] \\ &= e^2.\end{aligned}$$

[4] 3. We have

$$A = \int_{\frac{1}{4}}^{\frac{3}{4}} \frac{1}{16x^2 - 8x + 5}\,dx.$$

To evaluate the integral, we must complete the square:

$$\begin{aligned}16x^2 - 8x + 5 &= 16\left[x^2 - \frac{1}{2}x + \frac{5}{16}\right] \\ &= 16\left[\left(x^2 - \frac{1}{2}x + \frac{1}{16}\right) + \frac{5}{16} - \frac{1}{16}\right] \\ &= 16\left(x - \frac{1}{4}\right)^2 + 4 \\ &= (4x-1)^2 + 4.\end{aligned}$$

Thus

$$A = \int_{\frac{1}{4}}^{\frac{3}{4}} \frac{1}{(4x-1)^2 + 4}\,dx.$$

Now we let $u = 4x - 1$ so $\frac{1}{4}\,du = dx$. When $x = \frac{1}{4}$, $u = 0$ and when $x = \frac{3}{4}$, $u = 2$. The

integral becomes

$$
\begin{aligned}
A &= \frac{1}{4}\int_0^2 \frac{1}{u^2+4}\,du \\
&= \frac{1}{4}\cdot\frac{1}{2}\left[\arctan\left(\frac{u}{2}\right)\right]_0^2 \\
&= \frac{1}{8}[\arctan(1)-\arctan(0)] \\
&= \frac{1}{8}\left[\frac{\pi}{4}-0\right] \\
&= \frac{\pi}{32}.
\end{aligned}
$$