MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 5 **MATHEMATICS 1001** FALL 2019

SOLUTIONS

[6] 1. (a) We form a regular partition of $[2, 3]$ into n subintervals of length

$$\Delta x = \frac{3-2}{n} = \frac{1}{n}.$$

As the sample point, we choose

$$x_i = a_i = 2 + i \cdot \frac{1}{n} = 2 + \frac{i}{n}$$

so

$$f(x_i) = f\left(2 + \frac{i}{n}\right) = \frac{4i^3}{n^3} + \frac{27i^2}{n^2} + \frac{60i}{n} + 44.$$

Thus

$$\begin{aligned}
&\int_2^3 x^2(4x+3)\,dx \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{4i^3}{n^3} + \frac{27i^2}{n^2} + \frac{60i}{n} + 44\right] \cdot \frac{1}{n} \\
&= \lim_{n\to\infty} \left[\frac{4}{n^4}\sum_{i=1}^{n} i^3 + \frac{27}{n^3}\sum_{i=1}^{n} i^2 + \frac{60}{n^2}\sum i + \frac{44}{n}\sum_{i=1}^{n} 1\right] \\
&= \lim_{n\to\infty} \left[\frac{4}{n^4}\cdot\frac{n^2(n+1)^2}{4} + \frac{27}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} + \frac{60}{n^2}\cdot\frac{n(n+1)}{2} + \frac{44}{n}\cdot n\right] \\
&= 1 + 9 + 30 + 44 \\
&= 84.
\end{aligned}$$

[3] (b) Alternatively, we have

$$\begin{aligned}
\int_2^3 x^2(4x+3)\,dx &= \int_2^3 (4x^3 + 3x^2)\,dx \\
&= \left[x^4 + x^3\right]_2^3 \\
&= [3^4 + 3^3] - [2^4 + 2^3] \\
&= 84.
\end{aligned}$$

[6] 2. (a) We use integration by parts with $w = \ln(x)$ so $dw = \dfrac{1}{x}\,dx$, and $dv = \sqrt{x}\,dx$ so $v = \frac{2}{3}x^{\frac{3}{2}}$. Thus

$$\begin{aligned}
\int_1^e \sqrt{x}\ln(x)\,dx &= \left[\frac{2}{3}x^{\frac{3}{2}}\ln(x)\right]_1^e - \int_1^e \frac{2}{3}x^{\frac{3}{2}} \cdot \frac{1}{x}\,dx \\
&= \left[\frac{2}{3}x^{\frac{3}{2}}\ln(x)\right]_1^e - \frac{2}{3}\int_1^e x^{\frac{1}{2}}\,dx \\
&= \left[\frac{2}{3}x^{\frac{3}{2}}\ln(x) - \frac{4}{9}x^{\frac{3}{2}}\right]_1^e \\
&= \left[\frac{2}{3}e^{\frac{3}{2}}\ln(e) - \frac{4}{9}e^{\frac{3}{2}}\right] - \left[\frac{2}{3}\ln(1) - \frac{4}{9}\right] \\
&= \frac{2}{9}e^{\frac{3}{2}} + \frac{4}{9}.
\end{aligned}$$

[6] (b) We can complete the square:

$$\begin{aligned}
2x^2 + 6x + 9 &= 2\left[x^2 + 3x + \frac{9}{2}\right] \\
&= 2\left[\left(x^2 + 3x + \frac{9}{4}\right) + \frac{9}{2} - \frac{9}{4}\right] \\
&= 2\left[\left(x + \frac{3}{2}\right)^2 + \frac{9}{4}\right] \\
&= 2\left(x + \frac{3}{2}\right)^2 + \frac{9}{2}.
\end{aligned}$$

(In this case, we elect not to multiply the factor of 2 back into the expression in brackets, to avoid introducing a radical.) So now we have

$$\int_{-\frac{3}{2}}^0 \frac{1}{2x^2 + 6x + 9}\,dx = \int_{-\frac{3}{2}}^0 \frac{1}{2\left(x + \frac{3}{2}\right)^2 + \frac{9}{2}}\,dx = \frac{1}{2}\int_{-\frac{3}{2}}^0 \frac{1}{\left(x + \frac{3}{2}\right)^2 + \frac{9}{4}}\,dx.$$

Now we let $u = x + \frac{3}{2}$ so $du = dx$. When $x = -\frac{3}{2}$, $u = 0$. When $x = 0$, $u = \frac{3}{2}$. So the integral becomes

$$\int_{-\frac{3}{2}}^{0} \frac{1}{2x^2+6x+9}\,dx = \frac{1}{2}\int_0^{\frac{3}{2}} \frac{1}{u^2+\frac{9}{4}}\,du$$
$$= \frac{1}{2}\left[\frac{1}{\frac{3}{2}}\arctan\left(\frac{u}{\frac{3}{2}}\right)\right]_0^{\frac{3}{2}}$$
$$= \frac{1}{3}\left[\arctan\left(\frac{2u}{3}\right)\right]_0^{\frac{3}{2}}$$
$$= \frac{1}{3}[\arctan(1)-\arctan(0)]$$
$$= \frac{1}{3}\left[\frac{\pi}{4}-0\right]$$
$$= \frac{\pi}{12}.$$

[3] (c) Let $u=\cos(2t)$ so $du=-2\sin(2t)\,dt$ and $-\frac{1}{2}\,du=\sin(2t)\,dt$. When $t=0$, $u=\cos(0)=1$. When $t=\pi$, $u=\cos(2\pi)=1$. Thus the integral becomes

$$\int_0^{\pi} \sin(2t)\cos(2t)\sin(\cos(2t))\,dt = -\frac{1}{2}\int_1^1 u\sin(u)\,du.$$

We could now evaluate this integral using integration by parts, but it is much easier to simply observe that the bounds of integration are now the same, and therefore we can immediately conclude that

$$\int_0^{\pi} \sin(2t)\cos(2t)\sin(\cos(2t))\,dt = 0.$$

[6] (d) First we need to write the integrand as a piecewise function. Since $4-x^2=0$ when $x=\pm 2$, we need to see if $4-x^2$ is positive or negative for $x<-2$, for $-2\le x\le 2$, and for $x>2$. Checking a value of x on each of these three intervals, we have

$$|4-x^2| = \begin{cases} 4-x^2, & \text{for } -2\le x\le 2 \\ -(4-x^2), & \text{for } x<-2 \text{ and } x>2. \end{cases}$$

Now we have

$$\int_{-3}^{5}|4-x^2|\,dx = \int_{-3}^{-2}|4-x^2|\,dx + \int_{-2}^{2}|4-x^2|\,dx + \int_{2}^{5}|4-x^2|\,dx$$
$$= -\int_{-3}^{-2}(4-x^2)\,dx + \int_{-2}^{2}(4-x^2)\,dx - \int_{2}^{5}(4-x^2)\,dx$$
$$= -\left[4x-\frac{1}{3}x^3\right]_{-3}^{-2} + \left[4x-\frac{1}{3}x^3\right]_{-2}^{2} - \left[4x-\frac{1}{3}x^3\right]_{2}^{5}$$
$$= -\left(-\frac{7}{3}\right) + \frac{32}{3} - (-27)$$
$$= 40.$$

[5] 3. We can write

$$\begin{aligned}
\int_{-1}^{9} f(x)\,dx &= \int_{-1}^{1} f(x)\,dx + \int_{1}^{9} f(x)\,dx \\
&= \int_{-1}^{1} (-4)\,dx + \int_{1}^{9} \left(3\sqrt{x} - 7\right)\,dx \\
&= \Big[-4x\Big]_{-1}^{1} + \Big[2x^{\frac{3}{2}} - 7x\Big]_{1}^{9} \\
&= -8 + (-4) \\
&= -12.
\end{aligned}$$

[5] 4. We have

$$\begin{aligned}
A &= \int_{\frac{\pi}{2}}^{\pi} \tan\left(\frac{x}{3}\right)\,dx \\
&= \left[-3\ln\left|\cos\left(\frac{x}{3}\right)\right|\right]_{\frac{\pi}{2}}^{\pi} \\
&= -3\left[\ln\left|\cos\left(\frac{\pi}{3}\right)\right| - \ln\left|\cos\left(\frac{\pi}{6}\right)\right|\right] \\
&= -3\left[\ln\left(\frac{1}{2}\right) - \ln\left(\frac{\sqrt{3}}{2}\right)\right] \\
&= -3\ln\left(\frac{1}{\sqrt{3}}\right) \\
&= 3\ln\left(\sqrt{3}\right) \\
&= \frac{3}{2}\ln(3).
\end{aligned}$$