

MEMORIAL UNIVERSITY OF NEWFOUNDLAND
DEPARTMENT OF MATHEMATICS AND STATISTICS

DEFINITE INTEGRALS

Math 1001 Worksheet

FALL 2019

SOLUTIONS

1. (a) $\int_6^8 \frac{2}{(x-4)^2} dx$

(b) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^3(5x) dx$

2. (a) Using a regular partition and setting the sample point $x_i^* = x_i$, we can write

$$\int_0^2 \frac{x^3}{4} dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{x_i^3}{4} \Delta x.$$

Let

$$\Delta x = \frac{2-0}{n} = \frac{2}{n} \quad \text{and} \quad x_i = 0 + \frac{2i}{n} = \frac{2i}{n}$$

so

$$\frac{x_i^3}{4} = \frac{1}{4} \left(\frac{2i}{n} \right)^3 = \frac{2i^3}{n^3}.$$

Then

$$\begin{aligned} \int_0^2 \frac{x^3}{4} dx &= \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{2i^3}{n^3} \cdot \frac{2}{n} \\ &= \lim_{n \rightarrow \infty} \frac{4}{n^4} \sum_{i=1}^n i^3 \\ &= \lim_{n \rightarrow \infty} \frac{4}{n^4} \cdot \frac{n^2(n+1)^2}{4} \\ &= \lim_{n \rightarrow \infty} \left(1 + \frac{2}{n} + \frac{1}{n^2} \right) \\ &= 1. \end{aligned}$$

(b) Using a regular partition and setting the sample point $x_i^* = x_i$, we can observe that

$$\int_2^3 (2-7x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n (2-7x_i) \Delta x$$

where

$$\Delta x = \frac{3-2}{n} = \frac{1}{n} \quad \text{and} \quad x_i = 2 + \frac{i}{n}.$$

Now we have

$$2 - 7x_i = 2 - 7 \left(2 + \frac{i}{n} \right) = -\frac{7i}{n} - 12$$

and so

$$\begin{aligned} \int_2^3 (2 - 7x) dx &= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(-\frac{7i}{n} - 12 \right) \cdot \frac{1}{n} \\ &= \lim_{n \rightarrow \infty} \left(-\frac{7}{n^2} \sum_{i=1}^n i - \frac{12}{n} \sum_{i=1}^n 1 \right) \\ &= \lim_{n \rightarrow \infty} \left(-\frac{7}{n^2} \cdot \frac{n(n+1)}{2} - \frac{12}{n} \cdot n \right) \\ &= \lim_{n \rightarrow \infty} \left(-\frac{31}{2} - \frac{7}{2n} \right) \\ &= -\frac{31}{2}. \end{aligned}$$