

MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 3.2

Math 1001 Worksheet

FALL 2025

SOLUTIONS

1. (a) We extract a factor of $\cos(6x)$ and let $u = \sin(6x)$ so $\frac{1}{6} du = \cos(6x) dx$. Note that $x = \frac{\pi}{9}$ implies $u = \sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$, and $x = 0$ implies $u = 0$. Hence the integral becomes

$$\begin{aligned}\int_0^{\frac{\pi}{9}} \sin^2(6x) \cos^3(6x) dx &= \int_0^{\frac{\pi}{9}} \sin^2(6x) \cos^2(6x) \cos(6x) dx \\&= \int_0^{\frac{\pi}{9}} \sin^2(6x) [1 - \sin^2(6x)] \cos(6x) dx \\&= \frac{1}{6} \int_0^{\frac{\sqrt{3}}{2}} u^2 [1 - u^2] du \\&= \frac{1}{6} \int_0^{\frac{\sqrt{3}}{2}} [u^2 - u^4] du \\&= \frac{1}{6} \left[\frac{1}{3} u^3 - \frac{1}{5} u^5 \right]_0^{\frac{\sqrt{3}}{2}} \\&= \frac{11\sqrt{3}}{960}.\end{aligned}$$

- (b) We extract a factor of $\sin(x)$ and let $u = \cos(x)$ so $-du = \sin(x) dx$. The integral becomes

$$\begin{aligned}\int \sin^3(x) \cos^8(x) dx &= \int \sin^2(x) \cos^8(x) \sin(x) dx \\&= \int [1 - \cos^2(x)] \cos^8(x) \sin(x) dx \\&= - \int [1 - u^2] u^8 du \\&= \int [u^{10} - u^8] du \\&= \frac{1}{11} u^{11} - \frac{1}{9} u^9 + C \\&= \frac{1}{11} \cos^{11}(x) - \frac{1}{9} \cos^9(x) + C.\end{aligned}$$

(c) We extract a factor of $\cos(x)$ and let $u = \sin(x)$ so $du = \cos(x) dx$. The integral becomes

$$\begin{aligned}\int \sin^2(x) \cos^5(x) dx &= \int \sin^2(x) \cos^4(x) \cos(x) dx \\ &= \int \sin^2(x) [1 - \sin^2(x)]^2 \cos(x) dx \\ &= \int u^2 [1 - u^2]^2 du \\ &= \int [u^2 - 2u^4 + u^6] du \\ &= \frac{1}{3}u^3 - \frac{2}{5}u^5 + \frac{1}{7}u^7 + C \\ &= \frac{1}{3}\sin^3(x) - \frac{2}{5}\sin^5(x) + \frac{1}{7}\sin^7(x) + C.\end{aligned}$$

(d) First let $u = \ln(x)$ so $du = \frac{1}{x} dx$, so the integral becomes

$$\int \frac{\cos^3(\ln(x))}{x} dx = \int \cos^3(u) du = \int \cos^2(u) \cos(u) du = \int [1 - \sin^2(u)] \cos(u) du.$$

Now let $v = \sin(u)$ so $dv = \cos(u) du$ and we get

$$\begin{aligned}\int \frac{\cos^3(\ln(x))}{x} dx &= \int [1 - v^2] dv \\ &= v - \frac{1}{3}v^3 + C \\ &= \sin(u) - \frac{1}{3}\sin^3(u) + C \\ &= \sin(\ln(x)) - \frac{1}{3}\sin^3(\ln(x)) + C.\end{aligned}$$

(e) We begin by using integration by parts. Let $w = x$ so $dw = dx$, and let $dv = \sin^2(x) dx$. To find v , then, we use the half-angle formula and integrate:

$$v = \int \sin^2(x) dx = \frac{1}{2} \int [1 - \cos(2x)] dx = \frac{1}{2} \left[x - \frac{1}{2} \sin(2x) \right] = \frac{1}{2}x - \frac{1}{4} \sin(2x)$$

where, as usual, we have omitted the constant of integration. Hence

$$\begin{aligned}\int x \sin^2(x) dx &= \frac{1}{2}x^2 - \frac{1}{4}x \sin(2x) - \frac{1}{2} \int \left[x - \frac{1}{2} \sin(2x) \right] dx \\ &= \frac{1}{2}x^2 - \frac{1}{4}x \sin(2x) - \frac{1}{4}x^2 - \frac{1}{8} \cos(2x) + C \\ &= \frac{1}{4}x^2 - \frac{1}{8} \cos(2x) - \frac{1}{4}x \sin(2x) + C.\end{aligned}$$

(f) We simply have

$$\begin{aligned}
 \int \frac{1 - \tan^2(x)}{\sec^2(x)} dx &= \int \frac{1 - [\sec^2(x) - 1]}{\sec^2(x)} dx \\
 &= \int \frac{2 - \sec^2(x)}{\sec^2(x)} dx \\
 &= \int [2 \cos^2(x) - 1] dx \\
 &= \int [1 + \cos(2x) - 1] dx \\
 &= \int \cos(2x) dx \\
 &= \frac{1}{2} \sin(2x) + C.
 \end{aligned}$$

2. (a) We extract a factor of $\sec(x) \tan(x)$ and let $u = \sec(x)$ so $du = \sec(x) \tan(x) dx$. The integral becomes

$$\begin{aligned}
 \int \tan^5(x) \sec^5(x) dx &= \int \tan^4(x) \sec^4(x) \sec(x) \tan(x) dx \\
 &= \int [\sec^2(x) - 1]^2 \sec^4(x) \sec(x) \tan(x) dx \\
 &= \int [u^2 - 1]^2 u^4 du \\
 &= \int [u^8 - 2u^6 + u^4] du \\
 &= \frac{1}{9} u^9 - \frac{2}{7} u^7 + \frac{1}{5} u^5 + C \\
 &= \frac{1}{9} \sec^9(x) - \frac{2}{7} \sec^7(x) + \frac{1}{5} \sec^5(x) + C.
 \end{aligned}$$

(b) We rewrite the integral as

$$\begin{aligned}
 \int \frac{\cos^2(x)}{\sin^6(x)} dx &= \int \frac{1}{\sin^4(x)} \cdot \frac{\cos^2(x)}{\sin^2(x)} dx \\
 &= \int \csc^4(x) \cot^2(x) dx \\
 &= \int \csc^2(x) \cot^2(x) \csc^2(x) dx \\
 &= \int [1 + \cot^2(x)] \cot^2(x) \csc^2(x) dx.
 \end{aligned}$$

Now let $u = \cot(x)$ so $-du = \csc^2(x) dx$. The integral becomes

$$\begin{aligned}\int \frac{\cos^2(x)}{\sin^6(x)} dx &= - \int [1 + u^2] u^2 du \\ &= - \int [u^2 + u^4] du \\ &= - \left[\frac{1}{3} u^3 + \frac{1}{5} u^5 \right] + C \\ &= -\frac{1}{3} \cot^3(x) - \frac{1}{5} \cot^5(x) + C.\end{aligned}$$