MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

BASIC INTEGRATION | **Math 1001 Worksheet** | FALL 2019

SOLUTIONS

1. (a) We use u-substitution. Let $u = x^2 - 9$ so $du = 2x\,dx$ and $\frac{1}{2}\,du = x\,dx$. The integral becomes

$$\begin{aligned}\int \frac{x}{\sqrt{x^2-9}}\,dx &= \frac{1}{2}\int \frac{1}{\sqrt{u}}\,du \\ &= \frac{1}{2}\int u^{-\frac{1}{2}}\,du \\ &= \frac{1}{2}\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C \\ &= \sqrt{x^2-9} + C.\end{aligned}$$

(b) This is an arcsecant type integral with $k = 3$, so

$$\int \frac{1}{x\sqrt{x^2-9}}\,dx = \int \frac{1}{x\sqrt{x^2-3^2}}\,dx = \frac{1}{3}\operatorname{arcsec}\left(\frac{x}{3}\right) + C.$$

(c) We use integration by parts. Let $w = x$ so $dw = dx$, and let $dv = \csc^2(9x)\,dx$ so $v = -\frac{1}{9}\cot(9x)$. Then

$$\begin{aligned}\int x\csc^2(9x)\,dx &= -\frac{1}{9}x\cot(9x) + \frac{1}{9}\int \cot(9x)\,dx \\ &= -\frac{1}{9}x\cot(9x) + \frac{1}{9}\cdot\frac{1}{9}\ln|\sin(9x)| + C \\ &= -\frac{1}{9}x\cot(9x) + \frac{1}{81}\ln|\sin(9x)| + C.\end{aligned}$$

(d) We use u-substitution. Let $u = x^5$ so $du = 5x^4\,dx$ and $\frac{1}{5}\,du = x^4\,dx$. The integral becomes

$$\begin{aligned}\int x^4 e^{x^5}\,dx &= \frac{1}{5}\int e^u\,du \\ &= \frac{1}{5}e^u + C \\ &= \frac{1}{5}e^{x^5} + C.\end{aligned}$$

(e) We begin with u-substitution. Let $u = x^5$ so $du = 5x^4\,dx$ and $\frac{1}{5}\,du = x^4\,dx$. The integral becomes

$$\int x^9 e^{x^5}\,dx = \int x^5 e^{x^5}(x^4\,dx) = \frac{1}{5}\int u e^u\,du.$$

Now we use integration by parts, letting $w = u$ so $dw = du$, and $dv = e^u\,du$ so $v = e^u$. We obtain

$$\begin{aligned}\int x^9 e^{x^5}\,dx &= \frac{1}{5}\left[u e^u - \int e^u\,du\right]\\ &= \frac{1}{5}ue^u - \frac{1}{5}e^u + C\\ &= \frac{1}{5}x^5 e^{x^5} - \frac{1}{5}e^{x^5} + C.\end{aligned}$$

(f) We begin by completing the square:

$$\begin{aligned}9x^2 - 12x + 8 &= 9\left[\left(x^2 - \frac{4}{3}x\right) + \frac{8}{9}\right]\\ &= 9\left[\left(x^2 - \frac{4}{3}x + \frac{4}{9}\right) + \frac{8}{9} - \frac{4}{9}\right]\\ &= 9\left[\left(x - \frac{2}{3}\right)^2 + \frac{4}{9}\right]\\ &= (3x-2)^2 + 4.\end{aligned}$$

Thus the integral becomes

$$\int \frac{1}{9x^2 - 12x + 8}\,dx = \frac{1}{(3x-2)^2 + 4}\,dx.$$

Now let $u = 3x - 2$ so $du = 3\,dx$ and $\frac{1}{3}\,du = dx$. Finally,

$$\begin{aligned}\int \frac{1}{9x^2 - 12x + 8}\,dx &= \frac{1}{3}\int \frac{1}{u^2+4}\,du\\ &= \frac{1}{3}\cdot\frac{1}{2}\arctan\left(\frac{u}{2}\right) + C\\ &= \frac{1}{6}\arctan\left(\frac{3x-2}{2}\right) + C.\end{aligned}$$

(g) We try integration by parts, with $w = e^{4x}$ so $dw = 4e^{4x}\,dx$ and $dv = \cos(x)\,dx$ so $v = \sin(x)$. Then

$$\int e^{4x}\cos(x)\,dx = e^{4x}\sin(x) - 4\int e^{4x}\sin(x)\,dx.$$

We try integration by parts a second time. Again, we let $w = e^{4x}$ so $dw = 4e^{4x}\,dx$, and now we let $dv = \sin(x)\,dx$ so $v = -\cos(x)$. Thus

$$\begin{aligned}
\int e^{4x}\cos(x)\,dx &= e^{4x}\sin(x) - 4\left[-e^{4x}\cos(x) + 4\int e^{4x}\cos(x)\,dx\right] \\
&= e^{4x}\sin(x) + 4e^{4x}\cos(x) - 16\int e^{4x}\cos(x)\,dx \\
17\int e^{4x}\cos(x)\,dx &= e^{4x}\sin(x) + 4e^{4x}\cos(x) + C \\
\int e^{4x}\cos(x)\,dx &= \frac{1}{17}e^{4x}\sin(x) + \frac{4}{17}e^{4x}\cos(x) + C.
\end{aligned}$$

(h) We begin by performing long division:

$$\begin{array}{r}
6x - 1 \\
2x-5\,\big)\overline{12x^2 - 32x + 14} \\
\underline{12x^2 - 30x\phantom{{}+14}} \\
-2x + 14 \\
\underline{-2x + 5} \\
9
\end{array}$$

Now we can write

$$\begin{aligned}
\int \frac{12x^2 - 32x + 14}{2x - 5}\,dx &= \int \left(6x - 1 + \frac{9}{2x-5}\right)dx \\
&= 3x^2 - x + \frac{9}{2}\ln|2x - 5| + C.
\end{aligned}$$

(i) Let $u = \ln(x)$ so $du = \frac{1}{x}\,dx$. The integral becomes

$$\begin{aligned}
\int \frac{1}{x\sqrt{4 - \ln^2(x)}}\,dx &= \int \frac{1}{\sqrt{4 - u^2}}\,du \\
&= \arcsin\left(\frac{u}{2}\right) + C \\
&= \arcsin\left(\frac{\ln(x)}{2}\right) + C.
\end{aligned}$$

(j) Recall that $1 + \tan^2(x) = \sec^2(x)$, so

$$\int \cos^2(x)[1 + \tan^2(x)]\,dx = \cos^2(x)\sec^2(x)\,dx = \int dx = x + C.$$

Alternatively, we could write

$$\int \cos^2(x)[1 + \tan^2(x)]\,dx = \int [\cos^2(x) + \sin^2(x)]\,dx = \int dx = x + C.$$