MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

TEST 2 | **MATHEMATICS 1001** | FALL 2025

SOLUTIONS

[7] 1. (a) We use a regular partition with subintervals of width

$$\Delta x = \frac{3-1}{n} = \frac{2}{n}.$$

We choose the sample point

$$x_i^* = x_i = 1 + i\Delta x = 1 + \frac{2i}{n}.$$

Thus

$$\begin{aligned} f(x_i^*) &= \left(1 + \frac{2i}{n}\right)^2 + 2\left(1 + \frac{2i}{n}\right) - 3 \\ &= 1 + \frac{4i}{n} + \frac{4i^2}{n^2} + 2 + \frac{4i}{n} - 3 \\ &= \frac{4i^2}{n^2} + \frac{8i}{n}. \end{aligned}$$

Now we can write

$$\begin{aligned} \int_1^3 (x^2 + 2x - 3)\,dx &= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x \\ &= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{4i^2}{n^2} + \frac{8i}{n}\right) \cdot \frac{2}{n} \\ &= \lim_{n\to\infty} \left[\frac{8}{n^3} \sum_{i=1}^{n} i^2 + \frac{16}{n^2} \sum_{i=1}^{n} i\right] \\ &= \lim_{n\to\infty} \left[\frac{8}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} + \frac{16}{n^2} \cdot \frac{n(n+1)}{2}\right] \\ &= \frac{8}{3} + 8 \\ &= \frac{32}{3}. \end{aligned}$$

[3] (b) We have

$$\begin{aligned} \int_1^3 (x^2 + 2x - 3)\,dx &= \left[\frac{x^3}{3} + 2 \cdot \frac{x^2}{2} - 3x\right]_1^3 \\ &= (9 + 9 - 9) - \left(\frac{1}{3} + 1 - 3\right) \\ &= \frac{32}{3}. \end{aligned}$$

[5] 2. (a) We let $u = \sin\left(\frac{x}{4}\right)$ so $du = \frac{1}{4}\cos\left(\frac{x}{4}\right)\,dx$ and $4\,du = \cos\left(\frac{x}{4}\right)\,dx$. When $x = 0$, $u = \sin(0) = 0$. When $x = \pi$, $u = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$. Thus the integral becomes

$$\begin{aligned}\int_0^{\pi} \sin^3\left(\frac{x}{4}\right)\cos\left(\frac{x}{4}\right)\,dx &= 4\int_0^{\frac{\sqrt{2}}{2}} u^3\,du \\ &= 4\left[\frac{u^4}{4}\right]_0^{\frac{\sqrt{2}}{2}} \\ &= \frac{1}{4} - 0 \\ &= \frac{1}{4}.\end{aligned}$$

[5] (b) Observe that $2x - 1 = 0$ when $x = \frac{1}{2}$, so

$$|2x-1| = \begin{cases} 2x-1, & \text{for } x \geq \frac{1}{2} \\ -(2x-1), & \text{for } x < \frac{1}{2}. \end{cases}$$

Thus we can write

$$\begin{aligned}\int_{-1}^{1} |2x-1|\,dx &= \int_{-1}^{\frac{1}{2}} |2x-1|\,dx + \int_{\frac{1}{2}}^{1} |2x-1|\,dx \\ &= -\int_{-1}^{\frac{1}{2}} (2x-1)\,dx + \int_{\frac{1}{2}}^{1} (2x-1)\,dx \\ &= -\left[2\cdot\frac{x^2}{2} - x\right]_{-1}^{\frac{1}{2}} + \left[2\cdot\frac{x^2}{2} - x\right]_{\frac{1}{2}}^{1} \\ &= -\left[\left(\frac{1}{4} - \frac{1}{2}\right) - (1+1)\right] + \left[(1-1) - \left(\frac{1}{4} - \frac{1}{2}\right)\right] \\ &= \frac{5}{2}.\end{aligned}$$

[6] 3. (a) Since this is a partial rational function, we use the method of partial fractions. The decomposition is

$$\begin{aligned}\frac{3x^2 - 2x + 14}{(x-1)(x^2+4)} &= \frac{A}{x-1} + \frac{Bx + C}{x^2+4} \\ 3x^2 - 2x + 14 &= A(x^2+4) + (Bx+C)(x-1).\end{aligned}$$

When $x = 1$, we have $15 = 5A$ so $A = 3$. When $x = 0$, we have $14 = 4A - C$ so $C = 12 - 14 = -2$. When $x = -1$, we have $19 = 5A + 2B - 2C$ so $2B = 0$ and $B = 0$.

Thus the integral becomes

$$\int \frac{3x^2 - 2x + 14}{(x-1)(x^2+4)}\,dx = \int \left(\frac{3}{x-1} - \frac{2}{x^2+4} \right) dx$$
$$= 3\ln|x-1| - 2 \cdot \frac{1}{2} \arctan\left(\frac{x}{2}\right) + C$$
$$= 3\ln|x-1| - \arctan\left(\frac{x}{2}\right) + C.$$

[5] (b) Since there is an odd power of sine but an even power of cosine, we write

$$\int \sin^5(x)\cos^2(x)\,dx = \int \sin^4(x)\cos^2(x) \cdot \sin(x)\,dx$$
$$= \int [\sin^2(x)]^2 \cos^2(x) \cdot \sin(x)\,dx$$
$$= \int [1 - \cos^2(x)]^2 \cos^2(x) \cdot \sin(x)\,dx.$$

Now we let $u = \cos(x)$ so $du = -\sin(x)\,dx$ and $-\,du = \sin(x)\,dx$. the integral becomes

$$\int \sin^5(x)\cos^2(x) = -\int [1-u^2]^2\,du$$
$$= -\int (u^2 - 2u^4 + u^6)\,du$$
$$= -\left[\frac{u^3}{3} - 2 \cdot \frac{u^5}{5} + \frac{u^7}{7}\right] + C$$
$$= -\frac{1}{3}\cos^3(x) + \frac{2}{5}\cos^5(x) - \frac{1}{7}\cos^7(x) + C.$$

[9] 4. (a) The sketch of R is given in Figure 1. Note that the two curves intersect when

$$2 - x = \sqrt{x}$$
$$(2-x)^2 = x$$
$$4 - 4x + x^2 = x$$
$$x^2 - 5x + 4 = 0$$
$$(x-4)(x-1) = 0$$

that is, when $x = 1$ or $x = 4$. However, substitution of $x = 4$ back into the original equation demonstrates that it is a spurious solution. Thus the only intersection point occurs when $x = 1$ and, by substitution back into either function, we see that this is the point $(1, 1)$.

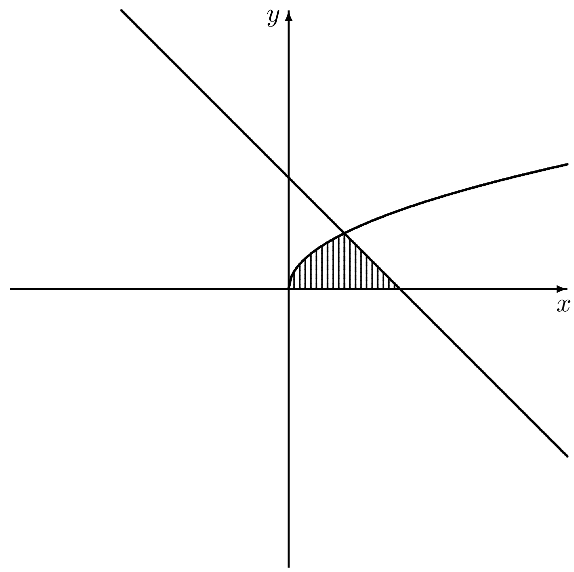

Figure 1: Question 4(a)

(b) The region is not vertically simple, but it can be split into vertically simple regions on the intervals $[0, 1]$ and $[1, 2]$. From the graph, we can see that the curve $y = 0$ is always the bottom boundary curve, while $y = \sqrt{x}$ is the top boundary curve on $[0, 1]$ and $y = 2 - x$ is the top boundary curve on $[1, 2]$. Thus

$$A = \int_0^1 \left(\sqrt{x} - 0\right)\,dx + \int_1^2 [(2 - x) - 0]\,dx$$
$$= \int_0^1 \sqrt{x}\,dx + \int_1^2 (2 - x)\,dx.$$

(c) The region is horizontally simple. The function $y = 2 - x$ can be written $x = 2 - y$ (the righthand boundary curve) while $y = \sqrt{x}$ becomes $x = y^2$ (the lefthand boundary curve). Hence

$$A = \int_0^1 [(2 - y) - y^2]\,dy$$
$$= \int_0^1 (2 - y - y^2)\,dy.$$