MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

Assignment 2

MATHEMATICS 1001

Fall 2025

SOLUTIONS

[3] 1. (a) Let $u = \sqrt{t}$ so $du = \frac{1}{2\sqrt{t}} dt$ and $2 du = \frac{1}{\sqrt{t}} dt$. The integral becomes

$$\int \frac{\tan(\sqrt{t})}{\sqrt{t}} dt = 2 \int \tan(u) du$$
$$= -2 \ln|\cos(u)| + C$$
$$= -2 \ln|\cos(\sqrt{t})| + C.$$

[4] (b) Observe that

$$\int \left(\frac{x}{\sin(x^3)}\right)^2 dx = \int \frac{x^2}{\sin^2(x^3)} dx.$$

Let $u = x^3$ so $du = 3x^2 dx$ and $\frac{1}{3} du = x^2 dx$. The integral becomes

$$\int \left(\frac{x}{\sin(x^3)}\right)^2 dx = \frac{1}{3} \int \frac{1}{\sin^2(u)} du$$
$$= \frac{1}{3} \int \csc^2(u) du$$
$$= \frac{1}{3} [-\cot(u)] + C$$
$$= -\frac{1}{2} \cot(x^3) + C.$$

[5] (c) Let $u = x^3 - 2$ so $du = 3x^2 dx$ and $\frac{1}{3} du = x^2 dx$. We can rewrite the integral as

$$\int x^8 (x^3 - 2)^7 dx = \int x^6 (x^3 - 2)^7 \cdot x^2 dx,$$

so we also need an expression for x^6 in terms of u. Since $u = x^3 - 2$, we have

$$x^3 = u + 2 \implies x^6 = (x^3)^2 = (u+2)^2$$

Now the integral becomes

$$\int x^8 (x^3 - 2)^7 dx = \frac{1}{3} \int (u + 2)^2 u^7 du$$

$$= \frac{1}{3} \int (u^9 + 4u^8 + 4u^7) du$$

$$= \frac{1}{3} \left[\frac{u^{10}}{10} + \frac{4u^9}{9} + \frac{4u^8}{8} \right] + C$$

$$= \frac{1}{30} u^{10} + \frac{4}{27} u^9 + \frac{1}{6} u^8 + C$$

$$= \frac{1}{30} (x^3 - 2)^{10} + \frac{4}{27} (x^3 - 2)^9 + \frac{1}{6} (x^3 - 2)^8 + C.$$

[3] 2. (a) Note that

$$\int \frac{3x^3 - 12x}{x^4 - 8x^2} \, dx = 3 \int \frac{x^3 - 4x}{x^4 - 8x^2} \, dx.$$

Let $u = x^4 - 8x^2$ so $du = (4x^3 - 16x) dx$ and $\frac{1}{4} du = (x^3 - 4x) dx$. Now we can write

$$\int \frac{3x^3 - 12x}{x^4 - 8x^2} dx = \frac{3}{4} \int \frac{1}{u} du$$
$$= \frac{3}{4} \ln|u| + C$$
$$= \frac{3}{4} \ln|x^4 - 8x^2| + C.$$

[5] (b) This is an improper rational function (because the numerator is a polynomial of degree 3, while the denominator is only a polynomial of degree 2), which suggests that we first try rewriting the function using long division. We have

$$\begin{array}{r}
x^{2} \\
x - 2 \overline{\smash{\big)}\,x^{3} - 2x^{2}} & + 1 \\
\underline{x^{3} - 2x^{2}} & 1
\end{array}$$

SO

$$\int \frac{x^3 - 2x^2 + 1}{x - 2} dx = \int \left(x^2 + \frac{1}{x - 2}\right) dx$$
$$= \frac{1}{3}x^3 + \ln|x - 2| + C.$$