# MEMORIAL UNIVERSITY OF NEWFOUNDLAND

## DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 1 | **MATHEMATICS 1001** | FALL 2019

## SOLUTIONS

[4] 1. (a) We can write the integral as

$$\int \left(5x + \sqrt[4]{x} - \frac{7}{x^7} - \frac{6}{x}\right) dx = 5\int x\,dx + \int x^{\frac{1}{4}}\,dx - 7\int x^{-7}\,dx - 6\int x^{-1}\,dx$$
$$= 5 \cdot \frac{x^2}{2} + \frac{x^{\frac{5}{4}}}{\frac{5}{4}} - 7 \cdot \frac{x^{-6}}{-6} - 6\ln|x| + C$$
$$= \frac{5}{2}x^2 + \frac{4}{5}x^{\frac{5}{4}} + \frac{7}{6x^6} - 6\ln|x| + C.$$

[3] (b) Since

$$\int \csc(x)\cot(x)\,dx = -\csc(x) + C,$$

we have

$$\int \csc(3x+8)\cot(3x+8)\,dx = -\frac{1}{3}\csc(3x+8) + C.$$

[4] (c) Since

$$\int \sinh(u)\,du = \cosh(u) + C \quad \text{and} \quad \int \sec^2(u)\,du = \tan(u) + C,$$

we have

$$\int [\sinh(4-u) + \sec^2(u-4)]\,du = \int \sinh(4-u)\,du + \int \sec^2(u-4)\,du$$
$$= \frac{\cosh(4-u)}{-1} + \frac{\tan(u-4)}{1} + C$$
$$= -\cosh(4-u) + \tan(u-4) + C.$$

[4] (d) Since

$$\int e^x\,dx = e^x + C \quad \text{and} \quad \int 5^x\,dx = \frac{5^x}{\ln(5)} + C,$$

we have

$$\int [e^{7x+3} - 5^{7x+3}]\,dx = \int e^{7x+3}\,dx - \int 5^{7x+3}\,dx$$
$$= \frac{e^{7x+3}}{7} - \frac{5^{7x+3}}{7\ln(5)} + C.$$

[3] (e) Since

$$\int x^9\,dx = \frac{x^{10}}{10} + C,$$

we have

$$\begin{aligned}\int \frac{(5-2x)^9}{3}\,dx &= \frac{1}{3}\int (5-2x)^9\,dx \\ &= \frac{1}{3}\cdot\frac{(5-2x)^{10}}{(-2)\cdot 10} + C \\ &= -\frac{1}{60}(5-2x)^{10} + C.\end{aligned}$$

[3] (f) We can rewrite this integral as

$$\int \frac{3}{(5-2x)^9}\,dx = 3\int (5-2x)^{-9}\,dx.$$

Now, since

$$\int x^{-9}\,dx = \frac{x^{-8}}{-8} + C,$$

we have

$$\begin{aligned}\int \frac{3}{(5-2x)^9}\,dx &= 3\cdot\frac{(5-2x)^{-8}}{-2\cdot(-8)} + C \\ &= \frac{3}{16(5-2x)^8} + C.\end{aligned}$$

[4] (g) Because this integral involves composition with a function that isn't linear (in fact, it's a polynomial of degree 9), our only approach is to multiply out:

$$\begin{aligned}\int \frac{(5-x^9)^2}{3}\,dx &= \frac{1}{3}\int (25 - 10x^9 + x^{18})\,dx \\ &= \frac{25}{3}\int dx - \frac{10}{3}\int x^9\,dx + \frac{1}{3}\int x^{18}\,dx \\ &= \frac{25}{3}x - \frac{10}{3}\cdot\frac{x^{10}}{10} + \frac{1}{3}\cdot\frac{x^{19}}{19} + C \\ &= \frac{25}{3}x - \frac{1}{3}x^{10} + \frac{1}{57}x^{19} + C.\end{aligned}$$

[4] (h) We can rewrite the integral as

$$\begin{aligned}\int \frac{e^x + e^{-x}}{e^{3x}}\,dx &= \int \left(\frac{e^x}{e^{3x}} + \frac{e^{-x}}{e^{3x}}\right)\,dx \\ &= \int e^{-2x}\,dx + \int e^{-4x}\,dx.\end{aligned}$$

Again, we know that

$$\int e^x\,dx = e^x + C$$

so

$$\begin{aligned}\int \frac{e^x + e^{-x}}{e^{3x}}\,dx &= \frac{e^{-2x}}{-2} + \frac{e^{-4x}}{-4} + C \\ &= -\frac{1}{2e^{2x}} - \frac{1}{4e^{4x}} + C.\end{aligned}$$

[4] (i) We can write

$$\begin{aligned}\int \cos(t)[\tan(t) - 4\sec(t)]\,dt &= \int \cos(t)\tan(t)\,dt - 4\int \cos(t)\sec(t)\,dt \\ &= \int \cos(t)\cdot\frac{\sin(t)}{\cos(t)}\,dt - 4\int \cos(t)\cdot\frac{1}{\cos(t)}\,dt \\ &= \int \sin(t)\,dt - 4\int dt \\ &= -\cos(t) - 4t + C.\end{aligned}$$

[4] 2. We can write

$$\begin{aligned}f(x) &= \frac{d}{dx}\left[\int f(x)\,dx\right] \\ &= \frac{d}{dx}\left[\ln\left(\frac{2x-3}{2x+3}\right) + C\right] \\ &= \frac{d}{dx}[\ln(2x-3) - \ln(2x+3) + C] \\ &= \frac{2}{2x-3} - \frac{2}{2x+3} \\ &= \frac{12}{4x^2-9}.\end{aligned}$$

[3] 3. We can write

$$\begin{aligned}f'(x) &= \frac{d}{dx}[f(x) + C] \\ &= \frac{d}{dx}\left[\int \sin(x^2)\,dx\right] \\ &= \sin(x^2).\end{aligned}$$