MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 4 | **MATHEMATICS 1001** | FALL 2019

Due: Friday, October 18th, 2019 at 4:00pm. SHOW ALL WORK.

> **Note:** You are encouraged to complete the "Worksheet on Area Under a Curve" before you work on this assignment.

1. Write out the first five terms of each sum. Find each sum (as a polynomial function of n) using the summation formulas.

   (a) $\displaystyle\sum_{i=1}^{n} i(i+1)$

   (b) $\displaystyle\sum_{i=1}^{n} (i^3 - 8i + 3)$

2. Consider the region R which lies under the graph of $f(x) = 4x - x^2$ and between the lines $x = 2$ and $x = 4$.

   (a) Determine the width Δx of each subinterval in a regular partition of $[2, 4]$.

   (b) Give an expression for the endpoint x_i^* of the ith subinterval. Use this to find formulas for the points m_i and M_i at which $f(x)$ attains its minimum and maximum values on the ith subinterval. (You may find it helpful to sketch the graph of $y = f(x)$.)

   (c) Determine $f(m_i)$ and $f(M_i)$.

   (d) Find formulas for the lower sum $s(n)$ and the upper sum $S(n)$.

   (e) Estimate the area A of the region R by finding the lower and upper sums with $n = 5$ subintervals. Give your answers to two decimal places.

   (f) Estimate A by finding the lower and upper sums with $n = 500$ subintervals. Give your answers to two decimal places.

   (g) Find the true value of A by taking the limit of the formulas you derived in part (d) as $n \to \infty$, and showing that these limits are equal.

3. Use the limit of a Riemann sum to determine the area A of each of the following regions. (In each case, use a regular partition and let the sample point $x_i^* = x_i$, that is, the right endpoint of the ith subinterval.)

   (a) The region under the curve $f(x) = x^3 + 5$ on the interval $[-1, 2]$.

   (b) The region under the curve $f(x) = (3x - 1)^2$ on the interval $[0, 5]$.

4. Consider the region R which lies under the curve $f(x) = x^2$ and between the y-axis and the line $x = b$. Use the limit of a Riemann sum to show that the area of R is given by $A = \dfrac{b^3}{3}$ for any $b > 0$.