SAMPLE FINAL

Please remember that each final exam is different, and the style and emphasis of the questions will vary from semester to semester.

- [7] 1. Use the definition of the definite integral as a limit of a sum to evaluate $\int_{-1}^{\frac{1}{2}} (9-4x^2) dx$.
 - 2. Find each of the following integrals:

[5] (a)
$$\int_{1}^{4} \frac{1}{\sqrt{x} (1+3\sqrt{x})^2} dx$$

5] (b)
$$\int \frac{20e^{2x}}{25+16e^{4x}} dx$$

[5] (c)
$$\int \frac{6x^3}{x^3 + 4} dx$$

[6] (d)
$$\int e^{7x} \sin(x) dx$$

3. Find each of the following integrals:

[7] (a)
$$\int \frac{\sin^5\left(\frac{x}{3}\right)}{\sqrt{\cos\left(\frac{x}{3}\right)}} dx$$

[7] (b)
$$\int \frac{\sqrt{x^2 - 9}}{x^3} dx$$

[7] (c)
$$\int \frac{7}{25x^2 - 20x + 13} dx$$

[7] (d)
$$\int \frac{5x^2}{(x+2)(x^2+1)} dx$$

[7] (e)
$$\int_{-\infty}^{0} x e^x dx$$

[10] 4. Sketch the region bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{1}{2}x$. Express each of the following as a single integral, but do <u>not</u> evaluate the integral:

- (a) the area of the region, as an integral with respect to x
- (b) the area of the region, as an integral with respect to y
- (c) the volume of the solid generated by revolving this region about the x-axis

[5] 5. Find and simplify the derivative of
$$f(x) = \int_{\sqrt{x}}^{3} \frac{6t^5}{\sqrt{4t^4 - 9}} dt$$
.

[5] 6. (a) Solve the initial value problem

$$ty\frac{dy}{dt} - t^3 + 1 = 0, \quad y(1) = 4.$$

- (b) The number of bacteria in a culture increases at a rate proportional to the number of bacteria present. If the number of bacteria in an initial culture reached 10 million after 2 weeks, and reached 40 million after 6 weeks, determine the number of bacteria in the initial culture.
- [6] (c) Consider the function

$$f(x) = \begin{cases} 6(x+3)^{-2}, & \text{for } 0 \le x \le 3\\ 0, & \text{otherwise.} \end{cases}$$

Explain why f(x) is a probability density function and use it to determine $P(0 \le X \le 1)$.

[6] 7. Use integration by parts to show that for any integer $n \neq 1$,

$$\int \sec^{n}(x) \, dx = \frac{\tan(x) \sec^{n-2}(x)}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2}(x) \, dx.$$

Use the reduction formula to evaluate $\int \sec^5(x) dx$.