Please remember that each final exam is different, and the style and emphasis of the questions will vary from semester to semester.

[7] 1. Use the definition of the definite integral as a limit of a sum to evaluate $\int_{-1}^{\frac{1}{2}} (9 - 4x^2)\, dx$.

2. Find each of the following integrals:

[5] (a) $\displaystyle\int_1^4 \frac{1}{\sqrt{x}\,(1 + 3\sqrt{x})^2}\, dx$

[5] (b) $\displaystyle\int \frac{20e^{2x}}{25 + 16e^{4x}}\, dx$

[5] (c) $\displaystyle\int \frac{6x^5}{x^3 + 4}\, dx$

[6] (d) $\displaystyle\int e^{7x} \sin(x)\, dx$

3. Find each of the following integrals:

[7] (a) $\displaystyle\int \frac{\sin^5\left(\frac{x}{3}\right)}{\sqrt{\cos\left(\frac{x}{3}\right)}}\, dx$

[7] (b) $\displaystyle\int \frac{\sqrt{x^2 - 9}}{x^3}\, dx$

[7] (c) $\displaystyle\int \frac{7}{25x^2 - 20x + 13}\, dx$

[7] (d) $\displaystyle\int \frac{5x^2}{(x + 2)(x^2 + 1)}\, dx$

[7] (e) $\displaystyle\int_{-\infty}^0 xe^x\, dx$

[10] 4. Sketch the region bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{1}{2}x$. Express each of the following as a single integral, but do <u>not</u> evaluate the integral:

(a) the area of the region, as an integral with respect to x

(b) the area of the region, as an integral with respect to y

(c) the volume of the solid generated by revolving this region about the x-axis

[5] 5. Find and simplify the derivative of $f(x) = \displaystyle\int_{\sqrt{x}}^3 \frac{6t^5}{\sqrt{4t^4 - 9}}\, dt$.

[5] 6. (a) Solve the initial value problem

$$ty\frac{dy}{dt} - t^3 + 1 = 0, \quad y(1) = 4.$$

[5] (b) The number of bacteria in a culture increases at a rate proportional to the number of bacteria present. If the number of bacteria in an initial culture reached 10 million after 2 weeks, and reached 40 million after 6 weeks, determine the number of bacteria in the initial culture.

[6] (c) Consider the function

$$f(x) = \begin{cases} 6(x+3)^{-2}, & \text{for } 0 \le x \le 3 \\ 0, & \text{otherwise.} \end{cases}$$

Explain why $f(x)$ is a probability density function and use it to determine $P(0 \le X \le 1)$.

[6] 7. Use integration by parts to show that for any integer $n \neq 1$,

$$\int \sec^n(x)\,dx = \frac{\tan(x)\sec^{n-2}(x)}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2}(x)\,dx.$$

Use the reduction formula to evaluate $\displaystyle\int \sec^5(x)\,dx$.