# MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 1.4 **Math 1000 Worksheet** FALL 2023

**For practice only. Not to be submitted.**

1. Use analytical methods to find the following limits. If a limit does not exist, explain why. Assign $\infty$ or $-\infty$ to the limit where appropriate.

(a) $\lim\limits_{x\to 4} \dfrac{2x^2-7x-4}{3x^2-14x+8}$

(b) $\lim\limits_{x\to -1} \dfrac{3x^2-9x-12}{x^3+7x^2+15x+9}$

(c) $\lim\limits_{t\to 2} \dfrac{t^2+t-6}{t^3-6t^2+12t-8}$

(d) $\lim\limits_{x\to \frac{1}{2}} \dfrac{3x}{2x-1}$

(e) $\lim\limits_{x\to -4} \dfrac{\sqrt{x+8}-2}{x+4}$

(f) $\lim\limits_{h\to 0} \dfrac{h^2-h}{\sqrt{h+3}-\sqrt{3}}$

(g) $\lim\limits_{x\to 3} \dfrac{x-5}{\sqrt{2x+3}+1}$

(h) $\lim\limits_{x\to 5} \dfrac{12(x+1)^{-1}-2}{x^2-6x+5}$

(i) $\lim\limits_{h\to 0} \dfrac{\frac{1}{h^2+9}-\frac{1}{9}}{h}$

(j) $\lim\limits_{x\to 0} \dfrac{\sin(8x)}{\sin(2x)}$

(k) $\lim\limits_{x\to 0} \dfrac{1-\cos^2(x)}{x}$

($\ell$) $\lim\limits_{x\to 0} \dfrac{\sin(3x^2)}{x\sin(x)}$

(m) $\lim\limits_{x\to \pi} \dfrac{\tan\left(\frac{x}{4}\right)}{1-\cos(x)}$

(n) $\lim\limits_{\theta\to 0} \dfrac{1-\sec(\theta)}{\theta\sec(\theta)}$

(o) $\lim\limits_{x\to 2} \dfrac{|x-2|-2}{x}$

(p) $\lim\limits_{x\to -2} \dfrac{|x-2|-2}{x}$

(q) $\lim\limits_{x\to 0} \dfrac{x^2-4x}{7x-|x|}$

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2. (a) Evaluate $\lim_{x\to 1} f(x)$ where

$$f(x) = \begin{cases} x^2 + 3x + 5, & \text{for } x \leq 1 \\ 7x - 2, & \text{for } x > 1 \end{cases}$$

(b) Evaluate $\lim_{x\to 1} g(x)$ where

$$g(x) = \begin{cases} x^2 + 3x + 5, & \text{for } x \leq 1 \\ 7x + 2, & \text{for } x > 1 \end{cases}$$

(c) Evaluate $\lim_{x\to 1} h(x)$ where

$$h(x) = \begin{cases} x^2 + 3x + 5, & \text{for } x \leq -3 \\ 7x - 2, & \text{for } x > -3 \end{cases}$$

3. Find the vertical asymptotes (if any) of the following functions. Justify your answers. For every vertical asymptote, use the lefthand and righthand limits to investigate whether $f(x)$ tends towards $\infty$ or $-\infty$ on either side of the asymptote.

(a) $f(x) = \dfrac{5x - 4 - x^2}{x^3 + 3x^2 - 9x + 5}$ (b) $f(x) = \dfrac{x^3 + 3x^2 - 9x + 5}{5x - 4 - x^2}$

4. Evaluate $\lim_{x\to \frac{\pi}{2}} \cot(x) \sin\left(\dfrac{1}{x}\right)$ using the Squeeze Theorem and the fact that

$$-1 \leq \sin\left(\frac{1}{x}\right) \leq 1.$$

5. Use the Squeeze Theorem to find $\lim_{x\to 0} x \cos\left(\dfrac{\pi}{2x}\right)$.