MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

Assignment 6 **MATHEMATICS 1000** Fall 2023

SOLUTIONS

[3] 1. (a) We use the Chain Rule twice:

$$\begin{aligned} f'(x) &= -\csc(\sin^5(x))\cot(\sin^5(x))[\sin^5(x)]' \\ &= -\csc(\sin^5(x))\cot(\sin^5(x))[5\sin^4(x)][\sin(x)]' \\ &= -\csc(\sin^5(x))\cot(\sin^5(x))[5\sin^4(x)][\cos(x)] \\ &= -5\cos(x)\sin^4(x)\csc(\sin^5(x))\cot(\sin^5(x)). \end{aligned}$$

[3] (b) We use the Product Rule, followed by the Chain Rule:

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx}[(x^2+1)^6](2x-3)^9 + \frac{d}{dx}[(2x-3)^9](x^2+1)^6 \\ &= \left[6(x^2+1)^5\frac{d}{dx}[x^2+1]\right](2x-3)^9 + \left[9(2x-3)^8\frac{d}{dx}[2x-3]\right](x^2+1)^6 \\ &= 6(x^2+1)^5(2x)(2x-3)^9 + 9(2x-3)^8(2)(x^2+1)^6 \\ &= 12x(x^2+1)^5(2x-3)^9 + 18(x^2+1)^6(2x-3)^8. \end{aligned}$$

Optionally, the derivative could be further simplified by factoring:

$$\begin{aligned} \frac{dy}{dx} &= 6(x^2+1)^5(2x-3)^8[2x(2x-3)+3(x^2+1)] \\ &= 6(x^2+1)^5(2x-3)^8(7x^2-6x+3). \end{aligned}$$

[3] (c) We use the Chain Rule, followed by the Product Rule:

$$\begin{aligned} f'(x) &= \sec^2(x^3 3^x)[x^3 3^x]' \\ &= \sec^2(x^3 3^x)[[x^3]'3^x + [3^x]'x^3] \\ &= \sec^2(x^3 3^x)[3x^2 3^x + x^3 3^x \ln(3)]. \end{aligned}$$

[4] (d) We use the Chain Rule three times:

$$\begin{aligned} \frac{dy}{dx} &= 10^{\cos(e^{7x})}\ln(10)\cdot\frac{d}{dx}[\cos(e^{7x})] \\ &= 10^{\cos(e^{7x})}\ln(10)\cdot[-\sin(e^{7x})]\cdot\frac{d}{dx}[e^{7x}] \\ &= 10^{\cos(e^{7x})}\ln(10)\cdot[-\sin(e^{7x})]\cdot e^{7x}\cdot\frac{d}{dx}[7x] \\ &= 10^{\cos(e^{7x})}\ln(10)\cdot[-\sin(e^{7x})]\cdot e^{7x}\cdot 7 \\ &= -7e^{7x}\sin(e^{7x})10^{\cos(e^{7x})}\ln(10). \end{aligned}$$

[7] 2. First we differentiate both sides of the equation with respect to x:

$$\begin{aligned}
[(x^2+y^2)^2]' &= [x^3-3xy^2]' \\
2(x^2+y^2)[x^2+y^2]' &= 3x^2-(3y^2+6xyy') \\
2(x^2+y^2)(2x+2yy') &= 3x^2-3y^2-6xyy' \\
4x^2yy'+4y^3y'+6xyy' &= 3x^2-3y^2-4x^3-4xy^2 \\
y'[4x^2y+4y^3+6xy] &= 3x^2-3y^2-4x^3-4xy^2 \\
y' &= \frac{3x^2-3y^2-4x^3-4xy^2}{4x^2y+4y^3+6xy}.
\end{aligned}$$

When $x = y = -\frac{1}{2}$, the slope of the tangent line is

$$y' = \frac{3\left(-\frac{1}{2}\right)^2 - 3\left(-\frac{1}{2}\right)^2 - 4\left(-\frac{1}{2}\right)^3 - 4\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)^2}{4\left(-\frac{1}{2}\right)^2\left(-\frac{1}{2}\right) + 4\left(-\frac{1}{2}\right)^3 + 6\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)} = \frac{1}{\frac{1}{2}} = 2.$$

Thus the equation of the tangent line has the form

$$\begin{aligned}
y &= 2x+b \\
-\frac{1}{2} &= 2\left(-\frac{1}{2}\right)+b \\
b &= \frac{1}{2},
\end{aligned}$$

so its equation is

$$y = 2x + \frac{1}{2}.$$