MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 5 **MATHEMATICS 1000** FALL 2023

SOLUTIONS

[3] 1. (a) We use the Quotient Rule:

$$\begin{aligned}\frac{dy}{dx} &= \frac{[e^x]'[\sin(x) - \pi^4] - e^x[\sin(x) - \pi^4]'}{[\sin(x) - \pi^4]^2} \\ &= \frac{e^x[\sin(x) - \pi^4] - e^x\cos(x)}{[\sin(x) - \pi^4]^2} \\ &= \frac{e^x\sin(x) - \pi^4 e^x - e^x\cos(x)}{[\sin(x) - \pi^4]^2}.\end{aligned}$$

[2] (b) We simply rewrite:

$$f(x) = \frac{\sin(x)}{\tan(x)} = \sin(x) \cdot \frac{\cos(x)}{\sin(x)} = \cos(x)$$

so that, clearly,

$$f'(x) = -\sin(x).$$

Note that we could use the Quotient Rule to obtain:

$$\begin{aligned}f'(x) &= \frac{[\sin(x)]'\tan(x) - \sin(x)[\tan(x)]'}{[\tan(x)]^2} = \frac{\cos(x)\tan(x) - \sin(x)\sec^2(x)}{\tan^2(x)} \\ &= \frac{\sin(x) - \sin(x)\sec^2(x)}{\tan^2(x)} = \frac{-\sin(x)[\sec^2(x) - 1]}{\tan^2(x)} \\ &= \frac{-\sin(x)\tan^2(x)}{\tan^2(x)} = -\sin(x),\end{aligned}$$

using the identity $\tan^2(x) + 1 = \sec^2(x)$, but obviously this is a much more onerous approach.

[4] (c) We use the Product Rule twice:

$$\begin{aligned}\frac{d}{dt}[g(t)] &= \frac{d}{dt}\left[\sqrt{t}\right]e^t\cos(t) + \sqrt{t}\frac{d}{dt}[e^t\cos(t)] \\ &= \frac{1}{2}t^{-\frac{1}{2}}e^t\cos(t) + \sqrt{t}\left(\frac{d}{dt}[e^t]\cos(t) + e^t\frac{d}{dt}[\cos(t)]\right) \\ &= \frac{e^t\cos(t)}{2\sqrt{t}} + \sqrt{t}[e^t\cos(t) - e^t\sin(t)] \\ &= \frac{e^t\cos(t)}{2\sqrt{t}} + \sqrt{t}e^t\cos(t) - \sqrt{t}e^t\sin(t).\end{aligned}$$

[4] (d) We use the Quotient Rule, followed by the Product Rule:

$$\begin{aligned} f'(x) &= \frac{[x\cot(x)]'(x^2+1) - x\cot(x)[x^2+1]'}{(x^2+1)^2} \\ &= \frac{([x]'\cot(x) + x[\cot(x)]')(x^2+1) - 2x^2\cot(x)}{(x^2+1)^2} \\ &= \frac{[\cot(x) - x\csc^2(x)](x^2+1) - 2x^2\cot(x)}{(x^2+1)^2} \\ &= \frac{\cot(x) - x^2\cot(x) - x^3\csc^2(x) - x\csc^2(x)}{(x^2+1)^2}. \end{aligned}$$

[7] 2. First note that

$$\begin{aligned} f'(x) &= [\sec(x)]'\tan(x) + \sec(x)[\tan(x)]' \\ &= \sec(x)\tan^2(x) + \sec^3(x) \\ f'\left(\frac{\pi}{3}\right) &= \sec\left(\frac{\pi}{3}\right)\tan^2\left(\frac{\pi}{3}\right) + \sec^3\left(\frac{\pi}{3}\right) \\ &= 2\left(\sqrt{3}\right)^2 + 2^3 \\ &= 14. \end{aligned}$$

Hence the slope of the tangent line is $m_T = 14$ and the slope of the normal line is $m_N = -\frac{1}{14}$. Furthermore, the point on the curve at which $x = \frac{\pi}{3}$ has y-coordinate

$$f\left(\frac{\pi}{3}\right) = \sec\left(\frac{\pi}{3}\right)\tan\left(\frac{\pi}{3}\right) = 2\sqrt{3}.$$

Therefore the tangent line has an equation of the form

$$\begin{aligned} y - 2\sqrt{3} &= 14\left(x - \frac{\pi}{3}\right) \\ y &= 14x + 2\sqrt{3} - \frac{14\pi}{3}. \end{aligned}$$

The normal line has an equation of the form

$$\begin{aligned} y - 2\sqrt{3} &= -\frac{1}{14}\left(x - \frac{\pi}{3}\right) \\ y &= -\frac{1}{14}x + 2\sqrt{3} + \frac{\pi}{42}. \end{aligned}$$