

MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 3.5

Math 1000 Worksheet

FALL 2023

SOLUTIONS

1. (a) We want an angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ whose sine is $-\frac{\sqrt{2}}{2}$, so

$$\arcsin\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}.$$

- (b) We want an angle between 0 and π whose cosine is $-\frac{\sqrt{2}}{2}$, so

$$\arccos\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}.$$

- (c) First note that

$$\operatorname{arcsec}\left(\frac{2\sqrt{3}}{3}\right) = \arccos\left(\frac{3}{2\sqrt{3}}\right) = \arccos\left(\frac{\sqrt{3}}{2}\right),$$

so we want an angle between 0 and π whose cosine is $\frac{\sqrt{3}}{2}$. Hence

$$\operatorname{arcsec}(\sqrt{2}) = \frac{\pi}{6}.$$

- (d) Note that $\frac{9\pi}{4} > \frac{\pi}{2}$ so we cannot simply use the cancellation equation. But $\tan\left(\frac{9\pi}{4}\right) = \tan\left(\frac{\pi}{4}\right) = 1$ and so

$$\arctan\left(\tan\left(\frac{9\pi}{4}\right)\right) = \frac{\pi}{4}.$$

- (e) Let $\theta = \arccos\left(\frac{5}{13}\right)$. Then we can construct a right triangle with an interior angle θ , adjacent sidelength 5 and hypotenuse of length 13. By the Pythagorean theorem, the remaining side has length

$$\sqrt{13^2 - 5^2} = \sqrt{144} = 12 \implies \sin\left(\arccos\left(\frac{5}{13}\right)\right) = \sin(\theta) = \frac{12}{13}.$$

- (f) Let $\theta = \arctan(2)$. We construct a right triangle with interior angle θ , opposite sidelength 2 and adjacent sidelength 1. Then the length of the hypotenuse is

$$\sqrt{1^2 + 2^2} = \sqrt{5} \implies \cos(\arctan(2)) = \cos(\theta) = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}.$$

$$2. (a) \quad y' = \frac{1}{\ln(x)\sqrt{[\ln(x)]^2 - 1}} \cdot [\ln(x)]' = \frac{1}{x \ln(x)\sqrt{[\ln(x)]^2 - 1}}$$

$$(b) \quad y' = (x^2)' \arctan(3x) + [\arctan(3x)]' x^2 \\ = 2x \arctan(3x) + \frac{1}{1+9x^2} \cdot (3x)' \cdot x^2 = 2x \arctan(3x) + \frac{3x^2}{1+9x^2}$$

$$(c) \quad y' = \frac{1}{\sqrt{1-\tan^2(t^2)}} \cdot [\tan(t^2)]' \\ = \frac{1}{\sqrt{1-\tan^2(t^2)}} \cdot \sec^2(t^2) \cdot (t^2)' = \frac{2t \sec^2(t^2)}{\sqrt{1-\tan^2(t^2)}}$$

(d) First we have

$$y' = \sec^2(\arcsin(t^2)) \cdot [\arcsin(t^2)]' = \sec^2(\arcsin(t^2)) \cdot \frac{1}{\sqrt{1-(t^2)^2}} \cdot (t^2)' \\ = \frac{2t \sec^2(\arcsin(t^2))}{\sqrt{1-t^4}}.$$

But note that if $\theta = \arcsin(t^2)$ then we can construct a right triangle with t^2 as the length of the side opposite θ , and 1 as the length of the hypotenuse. The length of the adjacent side must be

$$\sqrt{1^2 - (t^2)^2} = \sqrt{1-t^4}$$

so

$$\sec(\arcsin(t^2)) = \sec(\theta) = \frac{1}{\sqrt{1-t^4}} \quad \implies \quad \sec^2(\arcsin(t^2)) = \frac{1}{1-t^4}.$$

Thus

$$y' = \frac{2t \cdot \frac{1}{1-t^4}}{\sqrt{1-t^4}} = \frac{2t}{(1-t^4)^{\frac{3}{2}}}.$$

3. First, observe that

$$f'(x) = \frac{1}{\sqrt{1-\left(\frac{x-2}{2}\right)^2}} \cdot \frac{1}{2} - \frac{2}{\sqrt{1-\frac{x}{4}}} \cdot \frac{1}{4\sqrt{x}} = \frac{1}{2\sqrt{x-\frac{1}{4}x^2}} - \frac{1}{2\sqrt{x-\frac{1}{4}x^2}} = 0.$$

Thus the tangent line must be horizontal. When $x = 2$,

$$y = f(2) = \arcsin(0) - 2 \arcsin\left(\frac{\sqrt{2}}{2}\right) = 0 - 2\left(\frac{\pi}{4}\right) = -\frac{\pi}{2}.$$

Hence the equation of the tangent line is $y = -\frac{\pi}{2}$.

4. Differentiating implicitly, we have

$$\begin{aligned} \frac{d}{dx} [\sqrt{1-x^2y^2}] &= \frac{d}{dx} [\arccos(xy)] \\ \frac{1}{2\sqrt{1-x^2y^2}} \cdot \frac{d}{dx} [1-x^2y^2] &= -\frac{1}{\sqrt{1-(xy)^2}} \cdot \frac{d}{dx} [xy] \\ \frac{1}{2\sqrt{1-x^2y^2}} \left(-2xy^2 - 2x^2y \frac{dy}{dx} \right) &= -\frac{1}{\sqrt{1-x^2y^2}} \left(y + x \frac{dy}{dx} \right) \\ \left(-xy^2 - x^2y \frac{dy}{dx} \right) &= -y - x \frac{dy}{dx} \\ \frac{dy}{dx} (-x^2y + x) &= xy^2 - y \\ \frac{dy}{dx} &= \frac{xy^2 - y}{-x^2y + x} = \frac{y(xy - 1)}{-x(xy - 1)} = -\frac{y}{x}. \end{aligned}$$

5. Let $y = \arccos(x)$ so $\cos(y) = x$. Differentiating implicitly, we have

$$\begin{aligned} \frac{d}{dx} [\cos(y)] &= \frac{d}{dx} [x] \\ -\sin(y) \frac{dy}{dx} &= 1 \\ \frac{dy}{dx} &= -\frac{1}{\sin(y)}. \end{aligned}$$

Since $\sin^2(y) + \cos^2(y) = 1$, we know that $\sin(y) = \pm\sqrt{1 - \cos^2(y)}$. However, for $0 \leq y \leq \pi$, $\sin(y) \geq 0$, and so $\sin(y) = \sqrt{1 - \cos^2(y)} = \sqrt{1 - x^2}$. Hence

$$\frac{dy}{dx} = \frac{d}{dx} [\arccos(x)] = -\frac{1}{\sqrt{1-x^2}}.$$