

MEMORIAL UNIVERSITY OF NEWFOUNDLAND
DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 3.1

Math 1000 Worksheet

FALL 2023

SOLUTIONS

1. (a) Rewrite: $f(x) = e^2 e^x$

Differentiate: $f'(x) = e^2 \frac{d}{dx}[e^x] = e^2 e^x = e^{x+2}$

(b) Rewrite: $g(x) = 5 \sin(x) - \frac{1}{2} x^{\frac{1}{2}}$

Differentiate: $g'(x) = 5 \cos(x) - \frac{1}{2} \left(\frac{1}{2} x^{-\frac{1}{2}} \right) = 5 \cos(x) - \frac{1}{4\sqrt{x}}$

(c) $f'(t) = \frac{7}{3} t^{\frac{4}{3}} - [-\sin(t)] + 0 = \frac{7}{3} t^{\frac{4}{3}} + \sin(t)$

(d) $\frac{dy}{dx} = (2x^4)' \tan(x) + 2x^4 [\tan(x)]' = 8x^3 \tan(x) + 2x^4 \sec^2(x)$

(e) $g'(\theta) = [\sin(\theta)]' \tan(\theta) + \sin(\theta) [\tan(\theta)]' = \cos(\theta) \tan(\theta) + \sin(\theta) \sec^2(\theta)$

(f) Rewrite: $f(t) = t^{-1} \csc(t)$

Differentiate: $f'(t) = (t^{-1})' \csc(t) + t^{-1} [\csc(t)]'$
 $= -t^{-2} \csc(t) - t^{-1} \csc(t) \cot(t)$

$$= -\frac{\csc(t) + t \csc(t) \cot(t)}{t^2}$$

(g) $f'(x) = \frac{[1 - \sec(x)]'[1 + \sec(x)] - [1 - \sec(x)][1 + \sec(x)]'}{[1 + \sec(x)]^2}$
 $= \frac{-\sec(x) \tan(x)[1 + \sec(x)] - [1 - \sec(x)] \sec(x) \tan(x)}{[1 + \sec(x)]^2}$

$$= \frac{-2 \sec(x) \tan(x)}{[1 + \sec(x)]^2}$$

(h) $\frac{dy}{dx} = (x^3 e^x)' \cot(x) + x^3 e^x [\cot(x)]' = [(x^3)' e^x + x^3 (e^x)'] \cot(x) - x^3 e^x \csc^2(x)$

$$= 3x^2 e^x \cot(x) + x^3 e^x \cot(x) - x^3 e^x \csc^2(x)$$

(i) First we use the Quotient Rule:

$$f'(x) = \frac{(x e^x)'(\sqrt{x} - 3) - x e^x (\sqrt{x} - 3)'}{(\sqrt{x} - 3)^2}$$

Next we use the Product Rule to evaluate the first derivative in the numerator:

$$f'(x) = \frac{[(x)' e^x + x (e^x)'](\sqrt{x} - 3) - x e^x \cdot \frac{1}{2} x^{-\frac{1}{2}}}{(\sqrt{x} - 3)^2} = \frac{2(e^x + x e^x)(\sqrt{x} - 3) - \sqrt{x} e^x}{2(\sqrt{x} - 3)^2}$$

2. Differentiating, we have

$$f'(x) = 2 \sec^2(x) - \sqrt{2} \cos(x)$$

$$m_T = f' \left(\frac{\pi}{4} \right) = 2 \sec^2 \left(\frac{\pi}{4} \right) - \sqrt{2} \cos \left(\frac{\pi}{4} \right) = 2 \left(\sqrt{2} \right)^2 - \sqrt{2} \left(\frac{\sqrt{2}}{2} \right) = 3.$$

So then, using point-slope form, the equation of the tangent line is

$$y - 1 = 3 \left(x - \frac{\pi}{4} \right) \implies y = 3x - \frac{3\pi}{4} + 1.$$

The slope of the normal line must be

$$m_N = -\frac{1}{3}$$

and hence its equation is

$$y - 1 = -\frac{1}{3} \left(x - \frac{\pi}{4} \right) \implies y = -\frac{1}{3}x + \frac{\pi}{12} + 1.$$

3. Much like the proof that $\frac{d}{dx}[\sec(x)] = \sec(x) \tan(x)$, we write

$$\begin{aligned} \frac{d}{dx}[\csc(x)] &= \frac{d}{dx} \left[\frac{1}{\sin(x)} \right] = \frac{\frac{d}{dx}[1] \sin(x) - 1 \cdot \frac{d}{dx}[\sin(x)]}{\sin^2(x)} \\ &= \frac{(0) \sin(x) - 1 \cdot [\cos(x)]}{\sin^2(x)} = -\frac{\cos(x)}{\sin^2(x)} = -\frac{1}{\sin(x)} \cdot \frac{\cos(x)}{\sin(x)} \\ &= -\csc(x) \cot(x). \end{aligned}$$

4. Following the proof that $\frac{d}{dx}[\cot(x)] = -\csc^2(x)$, we have

$$\begin{aligned} \frac{d}{dx}[\tan(x)] &= \frac{d}{dx} \left[\frac{\sin(x)}{\cos(x)} \right] = \frac{\frac{d}{dx}[\sin(x)] \cos(x) - \sin(x) \frac{d}{dx}[\cos(x)]}{\cos^2(x)} \\ &= \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)} = \sec^2(x). \end{aligned}$$