MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

TEST 2 | **MATHEMATICS 1000-004** | FALL 2025

SOLUTIONS

[8] 1. By the limit definition of the derivative,

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{6(x+h)+7}-\sqrt{6x+7}}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{6x+6h+7}-\sqrt{6x+7}}{h} \cdot \frac{\sqrt{6x+6h+7}+\sqrt{6x+7}}{\sqrt{6x+6h+7}+\sqrt{6x+7}} \\
&= \lim_{h\to 0} \frac{(6x+6h+7)-(6x+7)}{h\left(\sqrt{6x+6h+7}+\sqrt{6x+7}\right)} \\
&= \lim_{h\to 0} \frac{6h}{h\left(\sqrt{6x+6h+7}+\sqrt{6x+7}\right)} \\
&= \lim_{h\to 0} \frac{6}{\sqrt{6x+6h+7}+\sqrt{6x+7}} \\
&= \frac{6}{\sqrt{6x+7}+\sqrt{6x+7}} \\
&= \frac{3}{\sqrt{6x+7}}.
\end{aligned}$$

[4] 2. One possibility is a function with a sharp corner (a cusp) at $x=0$, such as $f(x)=|x|$. Another possibility is a function with a vertical tangent line at $x=0$, such as $f(x)=\sqrt[3]{x}$.

[4] 3. (a) We use the Quotient Rule, follow by the Chain Rule:

$$\begin{aligned}
y' &= \frac{[1-e^{2x}]'(1+e^{2x})-(1-e^{2x})[1+e^{2x}]'}{(1+e^{2x})^2} \\
&= \frac{-e^{2x}\cdot[2x]'\cdot(1+e^{2x})-(1-e^{2x})\cdot e^{2x}\cdot[2x]'}{(1+e^{2x})^2} \\
&= \frac{-e^{2x}\cdot 2\cdot(1+e^{2x})-(1-e^{2x})\cdot e^{2x}\cdot 2}{(1+e^{2x})^2} \\
&= \frac{-2e^{2x}-2e^{4x}-2e^{2x}+2e^{4x}}{(1+e^{2x})^2} \\
&= \frac{-4e^{2x}}{(1+e^{2x})^2}.
\end{aligned}$$

[5] (b) We use the Product Rule twice:

$$\begin{aligned}
y' &= [x]'\tan(x)\sec(x) + x[\tan(x)\sec(x)]' \\
&= 1\cdot\tan(x)\sec(x) + x\left([\tan(x)]'\sec(x) + \tan(x)[\sec(x)]'\right) \\
&= \tan(x)\sec(x) + x(\sec^2(x)\cdot\sec(x) + \tan(x)\cdot\sec(x)\tan(x)) \\
&= \tan(x)\sec(x) + x\sec^3(x) + x\tan^2(x)\sec(x).
\end{aligned}$$

[5] (c) We use the Chain Rule twice:

$$\begin{aligned}
y' &= 3\sin^2\left(\frac{1}{x}\right)\cdot\left[\sin\left(\frac{1}{x}\right)\right]' \\
&= 3\sin^2\left(\frac{1}{x}\right)\cdot\cos\left(\frac{1}{x}\right)\cdot\left[\frac{1}{x}\right]' \\
&= 3\sin^2\left(\frac{1}{x}\right)\cdot\cos\left(\frac{1}{x}\right)\cdot\left(-\frac{1}{x^2}\right) \\
&= \frac{-3\sin^2\left(\frac{1}{x}\right)\cos\left(\frac{1}{x}\right)}{x^2}.
\end{aligned}$$

[5] (d) First we rewrite the function using the properties of logarithms:

$$\begin{aligned}
y &= \ln(x^7e^x) - \ln\left(\cot^2(x)\sqrt{x^4+1}\right) \\
&= [\ln(x^7) + \ln(e^x)] - [\ln(\cot^2(x)) + \ln((x^4+1)^{\frac{1}{2}})] \\
&= 7\ln(x) + x - 2\ln(\cot(x)) - \frac{1}{2}\ln(x^4+1) \\
y' &= 7\cdot\frac{1}{x} + 1 - 2\cdot\frac{1}{\cot(x)}[\cot(x)]' - \frac{1}{2}\cdot\frac{1}{x^4+1}[x^4+1]' \\
&= \frac{7}{x} + 1 - 2\cdot\frac{1}{\cot(x)}\cdot[-\csc^2(x)] - \frac{1}{2}\cdot\frac{1}{x^4+1}\cdot 4x^3 \\
&= \frac{7}{x} + 1 + \frac{2\csc^2(x)}{\cot(x)} - \frac{2x^3}{x^4+1}.
\end{aligned}$$

[7] 4. (a) We differentiate both sides of the equation with respect to x, using the Product Rule on

the righthand side:

$$\begin{aligned}
\frac{d}{dx}[\sin(x) + 4\cos(y)] &= \frac{d}{dx}[x^2 y] \\
\cos(x) + 4[-\sin(y)]\frac{dy}{dx} + \frac{d}{dx}[x^2]y + x^2\frac{d}{dx}[y] \\
\cos(x) - 4\sin(y)\frac{dy}{dx} &= 2xy + x^2\frac{dy}{dx} \\
\cos(x) - 2xy &= x^2\frac{dy}{dx} + 4\sin(y)\frac{dy}{dx} \\
\cos(x) - 2xy &= [x^2 + 4\sin(y)]\frac{dy}{dx} \\
\frac{dy}{dx} &= \frac{\cos(x) - 2xy}{x^2 + 4\sin(y)}.
\end{aligned}$$

[2] (b) The slope of the tangent line is

$$m = \frac{\cos(0) - 2 \cdot 0 \cdot \frac{\pi}{2}}{0^2 + 4\sin\left(\frac{\pi}{2}\right)} = \frac{1-0}{0+4} = \frac{1}{4}.$$

Hence the equation of the tangent line is

$$y - \frac{\pi}{2} = \frac{1}{4}(x - 0) \quad \Longrightarrow \quad \boxed{y = \frac{1}{4}x + \frac{\pi}{2}.}$$