MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

Test 1	**MATHEMATICS 1000-003**	Fall 2023

SOLUTIONS

[12] 1. (a) $f(2) = 1$

(b) $\lim_{x\to 2^-} f(x) = -2$

(c) $\lim_{x\to 2^+} f(x) = 1$

(d) $\lim_{x\to 2} f(x)$ does not exist (because the one-sided limits are not equal)

(e) $f(0)$ is undefined

(f) $\lim_{x\to 0^-} f(x) = -\infty$

(g) $\lim_{x\to 0^+} f(x) = -\infty$

(h) $\lim_{x\to 0} f(x) = -\infty$ (and therefore does not exist)

(i) $f(-3) = 0$

(j) $\lim_{x\to -3^-} f(x) = 4$

(k) $\lim_{x\to -3^+} f(x) = 4$

(l) $\lim_{x\to -3} f(x) = 4$

[4] 2. (a) Direct substitution produces a $\frac{0}{0}$ indeterminate form, so we use the Cancellation Method:

$$\begin{aligned}\lim_{x\to 4} \frac{2x^2 - 7x - 4}{3x^2 - 12x} &= \lim_{x\to 4} \frac{(2x+1)(x-4)}{3x(x-4)} \\ &= \lim_{x\to 4} \frac{2x+1}{3x} \\ &= \frac{9}{12} \\ &= \frac{3}{4}.\end{aligned}$$

[6] (b) Direct substitution yields a $\frac{0}{0}$ indeterminate form, so we use the Rationalisation Method:

$$\begin{aligned}
\lim_{x\to 2}\frac{1-\sqrt{7-3x}}{x^2-4} &= \lim_{x\to 2}\frac{1-\sqrt{7-3x}}{x^2-4}\cdot\frac{1+\sqrt{7-3x}}{1+\sqrt{7-3x}}\\
&= \lim_{x\to 2}\frac{1-(7-3x)}{(x^2-4)(1+\sqrt{7-3x})}\\
&= \lim_{x\to 2}\frac{3x-6}{(x^2-4)(1+\sqrt{7-3x})}\\
&= \lim_{x\to 2}\frac{3(x-2)}{(x-2)(x+2)(1+\sqrt{7-3x})}\\
&= \lim_{x\to 2}\frac{3}{(x+2)(1+\sqrt{7-3x})}\\
&= \frac{3}{4\cdot 2}\\
&= \frac{3}{8}.
\end{aligned}$$

[6] (c) Direct substitution results in a $\frac{0}{0}$ indeterminate form, so we rewrite the given function as

$$\begin{aligned}
\lim_{x\to -3}\frac{(x+4)^{-1}+3x^{-1}}{x+3} &= \lim_{x\to -3}\frac{\frac{1}{x+4}+\frac{3}{x}}{x+3}\\
&= \lim_{x\to -3}\frac{\frac{1}{x+4}\cdot\frac{x}{x}+\frac{3}{x}\cdot\frac{x+4}{x+4}}{x+3}\\
&= \lim_{x\to -3}\frac{4x+12}{x(x+4)}\cdot\frac{1}{x+3}\\
&= \lim_{x\to -3}\frac{4(x+3)}{x(x+4)}\cdot\frac{1}{x+3}\\
&= \lim_{x\to -3}\frac{4}{x(x+4)}\\
&= \frac{4}{-3\cdot 1}\\
&= -\frac{4}{3}.
\end{aligned}$$

[2] (d) We simply use direct substitution:

$$\lim_{x\to\frac{\pi}{2}}\frac{6-\cos(2x)}{2\cos(x)+1} = \frac{6-\cos(\pi)}{2\cos\left(\frac{\pi}{2}\right)+1} = \frac{6-(-1)}{2\cdot 0+1} = 7.$$

[4] (e) Direct substitution yields a $\frac{0}{0}$ indeterminate form, but we can rewrite the given limit as

$$\begin{aligned}
\lim_{x\to 0}\frac{\sin(x)}{\sin(3x)} &= \lim_{x\to 0}\frac{\sin(x)}{\sin(3x)}\cdot\frac{x}{x}\\
&= \lim_{x\to 0}\frac{\sin(x)}{x}\cdot\lim_{x\to 0}\frac{x}{\sin(3x)}\\
&= 1\cdot\lim_{x\to 0}\frac{x}{\sin(3x)}\cdot\frac{3}{3}\\
&= \frac{1}{3}\lim_{x\to 0}\frac{3x}{\sin(3x)}\\
&= \frac{1}{3}\cdot 1\\
&= \frac{1}{3}.
\end{aligned}$$

[6] (f) We use the fact that

$$|x| = \begin{cases} x & \text{for } x \geq 0 \\ -x & \text{for } x < 0 \end{cases}$$

and consider the one-sided limits. From the left,

$$\lim_{x\to 0^-}\frac{6x}{|x|+2x} = \lim_{x\to 0^-}\frac{6x}{(-x)+2x} = \lim_{x\to 0^-}\frac{6x}{x} = \lim_{x\to 0^-} 6 = 6.$$

From the right,

$$\lim_{x\to 0^+}\frac{6x}{|x|+2x} = \lim_{x\to 0^+}\frac{6x}{x+2x} = \lim_{x\to 0^+}\frac{6x}{3x} = \lim_{x\to 0^+} 2 = 2.$$

Since the one-sided limits are not equal, we must conclude that $\lim_{x\to 0}\frac{6x}{|x|+2x}$ does not exist.