

SOLUTIONS

- [12] 1. (a) $f(0)$ is undefined
- (b) $\lim_{x \rightarrow 0^-} f(x) = -\infty$
- (c) $\lim_{x \rightarrow 0^+} f(x) = -\infty$
- (d) $\lim_{x \rightarrow 0} f(x) = -\infty$ (and therefore does not exist)
- (e) $f(-3) = 0$
- (f) $\lim_{x \rightarrow -3^-} f(x) = 4$
- (g) $\lim_{x \rightarrow -3^+} f(x) = 4$
- (h) $\lim_{x \rightarrow -3} f(x) = 4$
- (i) $f(2) = 1$
- (j) $\lim_{x \rightarrow 2^-} f(x) = -2$
- (k) $\lim_{x \rightarrow 2^+} f(x) = 1$
- (l) $\lim_{x \rightarrow 2} f(x)$ does not exist (because the one-sided limits are not equal)
- [4] 2. (a) Direct substitution produces a $\frac{0}{0}$ indeterminate form, so we use the Cancellation Method:

$$\begin{aligned}\lim_{x \rightarrow 4} \frac{2x^2 - 7x - 4}{3x^2 - 12x} &= \lim_{x \rightarrow 4} \frac{(2x + 1)(x - 4)}{3x(x - 4)} \\ &= \lim_{x \rightarrow 4} \frac{2x + 1}{3x} \\ &= \frac{9}{12} \\ &= \frac{3}{4}.\end{aligned}$$

[6] (b) Direct substitution yields a $\frac{0}{0}$ indeterminate form, so we use the Rationalisation Method:

$$\begin{aligned}\lim_{x \rightarrow 2} \frac{1 - \sqrt{7 - 3x}}{x^2 - 4} &= \lim_{x \rightarrow 2} \frac{1 - \sqrt{7 - 3x}}{x^2 - 4} \cdot \frac{1 + \sqrt{7 - 3x}}{1 + \sqrt{7 - 3x}} \\ &= \lim_{x \rightarrow 2} \frac{1 - (7 - 3x)}{(x^2 - 4)(1 + \sqrt{7 - 3x})} \\ &= \lim_{x \rightarrow 2} \frac{3x - 6}{(x^2 - 4)(1 + \sqrt{7 - 3x})} \\ &= \lim_{x \rightarrow 2} \frac{3(x - 2)}{(x - 2)(x + 2)(1 + \sqrt{7 - 3x})} \\ &= \lim_{x \rightarrow 2} \frac{3}{(x + 2)(1 + \sqrt{7 - 3x})} \\ &= \frac{3}{4 \cdot 2} \\ &= \frac{3}{8}.\end{aligned}$$

[6] (c) Direct substitution results in a $\frac{0}{0}$ indeterminate form, so we rewrite the given function as

$$\begin{aligned}\lim_{x \rightarrow -3} \frac{(x + 4)^{-1} + 3x^{-1}}{x + 3} &= \lim_{x \rightarrow -3} \frac{\frac{1}{x+4} + \frac{3}{x}}{x + 3} \\ &= \lim_{x \rightarrow -3} \frac{\frac{1}{x+4} \cdot \frac{x}{x} + \frac{3}{x} \cdot \frac{x+4}{x+4}}{x + 3} \\ &= \lim_{x \rightarrow -3} \frac{4x + 12}{x(x + 4)} \cdot \frac{1}{x + 3} \\ &= \lim_{x \rightarrow -3} \frac{4(x + 3)}{x(x + 4)} \cdot \frac{1}{x + 3} \\ &= \lim_{x \rightarrow -3} \frac{4}{x(x + 4)} \\ &= \frac{4}{-3 \cdot 1} \\ &= -\frac{4}{3}.\end{aligned}$$

[2] (d) We simply use direct substitution:

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{6 - \cos(2x)}{2 \cos(x) + 1} = \frac{6 - \cos(\pi)}{2 \cos\left(\frac{\pi}{2}\right) + 1} = \frac{6 - (-1)}{2 \cdot 0 + 1} = 7.$$

[4] (e) Direct substitution yields a $\frac{0}{0}$ indeterminate form, but we can rewrite the given limit as

$$\begin{aligned}\lim_{x \rightarrow 0} \frac{\sin(x)}{\sin(3x)} &= \lim_{x \rightarrow 0} \frac{\sin(x)}{\sin(3x)} \cdot \frac{x}{x} \\ &= \lim_{x \rightarrow 0} \frac{\sin(x)}{x} \cdot \lim_{x \rightarrow 0} \frac{x}{\sin(3x)} \\ &= 1 \cdot \lim_{x \rightarrow 0} \frac{x}{\sin(3x)} \cdot \frac{3}{3} \\ &= \frac{1}{3} \lim_{x \rightarrow 0} \frac{3x}{\sin(3x)} \\ &= \frac{1}{3} \cdot 1 \\ &= \frac{1}{3}.\end{aligned}$$

[6] (f) We use the fact that

$$|x| = \begin{cases} x & \text{for } x \geq 0 \\ -x & \text{for } x < 0 \end{cases}$$

and consider the one-sided limits. From the left,

$$\lim_{x \rightarrow 0^-} \frac{6x}{|x| + 2x} = \lim_{x \rightarrow 0^-} \frac{6x}{(-x) + 2x} = \lim_{x \rightarrow 0^-} \frac{6x}{x} = \lim_{x \rightarrow 0^-} 6 = 6.$$

From the right,

$$\lim_{x \rightarrow 0^+} \frac{6x}{|x| + 2x} = \lim_{x \rightarrow 0^+} \frac{6x}{x + 2x} = \lim_{x \rightarrow 0^+} \frac{6x}{3x} = \lim_{x \rightarrow 0^+} 2 = 2.$$

Since the one-sided limits are not equal, we must conclude that $\lim_{x \rightarrow 0} \frac{6x}{|x| + 2x}$ does not exist.