MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

Section 1.3	**Math 1000 Worksheet**	Fall 2023

SOLUTIONS

1. (a) Using Basic Limit Property #2,

$$\lim_{x\to p}[f(x) - g(x)] = \lim_{x\to p} f(x) - \lim_{x\to p} g(x) = -5 - 4 = -9.$$

(b) Using Basic Limit Property #s 2 and 3,

$$\lim_{x\to p}[g(x) - 2f(x)] = \lim_{x\to p} g(x) - 2\lim_{x\to p} f(x) = 4 - 2(-5) = 14.$$

(c) Using Basic Limit Property #5,

$$\lim_{x\to p} \frac{f(x)}{g(x)} = \frac{\lim\limits_{x\to p} f(x)}{\lim\limits_{x\to p} g(x)} = \frac{-5}{4} = -\frac{5}{4}.$$

(d) Using Basic Limit Property #4 and the property for radicals of functions,

$$\lim_{x\to p} f(x)\sqrt{g(x)} = \lim_{x\to p} f(x) \cdot \sqrt{\lim_{x\to p} g(x)} = -5 \cdot \sqrt{4} = -10.$$

2. (a) By direct substitution, $\lim\limits_{x\to 5}(x^2 - 9x - 3) = 5^2 - 9(5) + 3 = 25 - 45 + 3 = -17.$

(b) By direct substitution, $\lim\limits_{x\to -3} \dfrac{\sqrt{1-x}}{x} = \dfrac{\sqrt{4}}{-3} = -\dfrac{2}{3}.$

(c) By direct substitution, $\lim\limits_{h\to 0} \dfrac{\cos(h)}{2^h} = \dfrac{\cos(0)}{2^0} = 1.$

(d) Observe that

$$|x-2| = \begin{cases} x-2 & \text{for } x \geq 2 \\ -(x-2) & \text{for } x < 2. \end{cases}$$

Since this is a piecewise function which changes definition at $x = 2$, we must evaluate both the lefthand and righthand limits as $x \to 2$.

For the lefthand limit (where $x < 2$), we can write $|x-2| = -(x-2)$, giving

$$\lim_{x\to 2^-} \frac{|x-2|}{x-2} = \lim_{x\to 2^-} \frac{-(x-2)}{x-2} = \lim_{x\to 2^-} -1 = -1.$$

For the righthand limit (where $x > 2$), we can write $|x-2| = x-2$, giving

$$\lim_{x\to 2^+} \frac{|x-2|}{x-2} = \lim_{x\to 2^+} \frac{x-2}{x-2} = \lim_{x\to 2^+} 1 = 1.$$

Since the one-sided limits are not equal, we can conclude that $\lim\limits_{x\to 2} \dfrac{|x-2|}{x-2}$ does not exist.

3. (a) Although $f(x)$ is a piecewise function, its definition does not change at $x = \frac{\pi}{6}$, so we can use direct substitution. Since $f(x) = \cos(x)$ for all $x \leq 0$,

$$\lim_{x \to -\frac{\pi}{6}} f(x) = \lim_{x \to -\frac{\pi}{6}} \cos(x) = \cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}.$$

(b) Since $f(x)$ changes its definition at $x = 0$, we must consider the one-sided limits. Immediately to the left of $x = 0$, $f(x) = \cos(x)$ so

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \cos(x) = \cos(0) = 1.$$

Immediately to the right of $x = 0$, $f(x) = 1 - 4x$ so

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (1 - 4x) = 1 - 4(0) = 1.$$

Since these are in agreement,

$$\lim_{x \to 0} f(x) = 1.$$

(c) Again, because $f(x)$ changes its definition at $x = 3$, we must calculate the one-sided limits. Immediately to the left of $x = 3$, $f(x) = 1 - 4x$ so

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (1 - 4x) = 1 - 4(3) = -11.$$

Immediately to the right of $x = 3$, $f(x) = \frac{9}{x}$ so

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} \frac{9}{x} = \frac{9}{3} = 3.$$

Since the one-sided limits are not equal, $\lim_{x \to 3} f(x)$ does not exist.

4. Since this is a piecewise function whose behaviour changes at $x = -2$, we must check the one-sided limits:

$$\lim_{x \to -2^-} f(x) = \lim_{x \to -2^-} k^2 x = -2k^2$$

and

$$\lim_{x \to -2^+} f(x) = \lim_{x \to -2^+} (4k - x) = 4k + 2.$$

If the limit exists, then these one-sided limits must be equal, so we set

$$\begin{aligned} -2k^2 &= 4k + 2 \\ 2k^2 + 4k + 2 &= 0 \\ 2(k+1)^2 &= 0, \end{aligned}$$

and hence $k = -1.$

(Note that the second part of the definition of $f(x)$ did not affect our workings because it applies only for $x = -2$, which has no effect on the limit as x approaches -2.)