

## MATHEMATICS 1000 (Calculus I)

### Related Rates Examples and Basic Geometrical Formulae

The following problems will be covered during regular lectures.

1. A spherical soap bubble is absorbing  $10 \text{ cm}^3$  of air every second. How quickly is the radius of the bubble increasing at the moment when it measures 1 cm?
2. A 25-foot ladder is leaning against the exterior wall of a house. The base of the ladder is pulled away from the wall at a rate of 2 feet per second. How fast is the top of the ladder moving down the wall when the base of the ladder is 7 feet from the wall?
3. An inverted cone is being filled with yoghurt. The cone is 12 cm high and has a radius of 4 cm at the top. If the volume of yoghurt in the cone is increasing at a rate of  $0.4 \text{ cm}^3/\text{sec}$ , determine how quickly the yoghurt is rising at the instant when it is 2 cm deep.
4. Marshall Wyatt Earp is in hot pursuit of notorious outlaw Johnny Ringo. At 9:00am, Earp is 200 km west of Ringo's position, and is riding east at a constant speed of 50 km/hr. Ringo, having been tipped off by the Clanton gang, starts heading north at 40 km/hr. To one decimal place, how quickly is the distance between them changing at high noon?

The following problems will be covered during Tutorial #7.

1. A cylindrical water tank with a 40-metre diameter is draining, and the level of water inside is decreasing at a constant rate of 1.5 m/min. How fast is the volume of water decreasing?
2. A weather balloon rises vertically, and is being observed from a point on the ground that is 50 metres from the spot directly beneath the balloon. At what rate is the balloon rising when the angle between the ground and the observer's line of sight is  $45^\circ$  and is increasing at  $3^\circ$  per second?

right triangle (legs $a$ , $b$ , hypotenuse $c$ )	area	$A = \frac{1}{2}ab$
	perimeter	$P = a + b + c$
rectangle (length $\ell$ , width $w$ )	area	$A = \ell w$
	perimeter	$P = 2\ell + 2w$
square (sidelength $\ell$ )	area	$A = \ell^2$
	perimeter	$P = 4\ell$
circle (radius $r$ )	area	$A = \pi r^2$
	circumference	$C = 2\pi r$

Table 1: Basic two-dimensional geometrical formulas.

rectangular prism (length $\ell$ , width $w$ , height $h$ )	volume	$V = \ell wh$
	surface area	$S = 2[\ell w + wh + \ell h]$
cube (sidelength $\ell$ )	volume	$V = \ell^3$
	surface area	$S = 6\ell^2$
sphere (radius $r$ )	volume	$V = \frac{4}{3}\pi r^3$
	surface area	$S = 4\pi r^2$
(right circular) cylinder (radius $r$ , height $h$ )	volume	$V = \pi r^2 h$
	surface area	$S = 2\pi r^2 + 2\pi r h$
(right circular) cone (radius $r$ , height $h$ )	volume	$V = \frac{1}{3}\pi r^2 h$
	surface area	$S = \pi r^2 + \pi r \sqrt{r^2 + h^2}$

Table 2: Basic three-dimensional geometrical formulas.