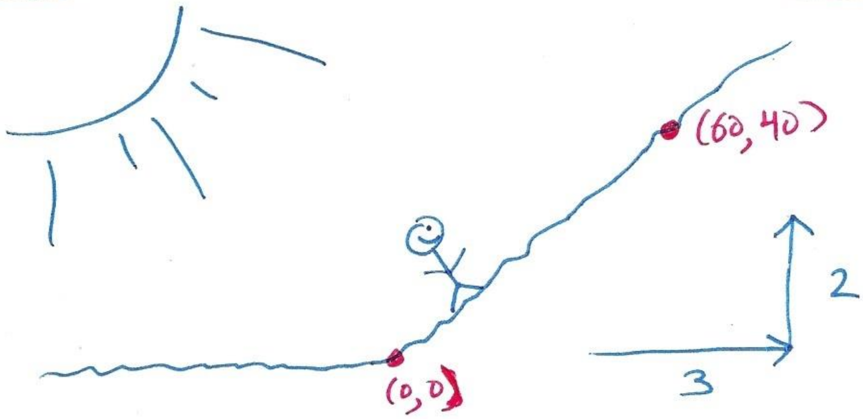
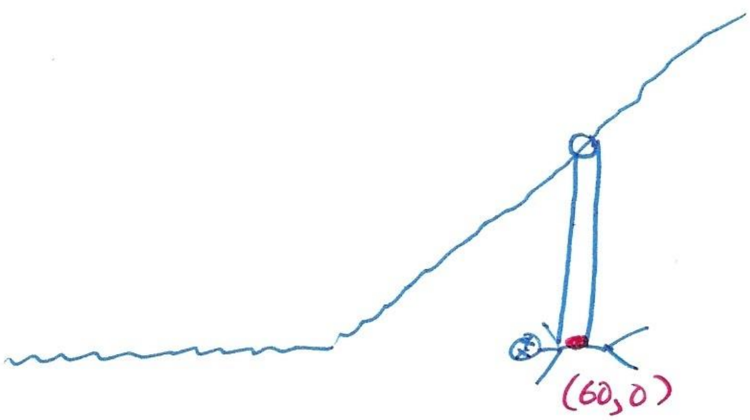
Section 1.1: The Limit of a Function



Suppose you are walking up a hill which is inclined such that, for every 3 metres your horizontal position changes, your vertical position changes by 2 metres.

Let x be your horizontal postion, and y be your vertical position.

Now we can assume that, when $x = 60$, then $y = 40$.



But now suppose that there is a pit dug into the hill such that, when $x = 60$, then $y = 0$.

Here we would say that, if the hill were a function, then the <u>value</u> of the function at $x = 60$ is 0. However, the <u>limit</u> of the function as x <u>approaches</u> 60 is 40.

Mathematically, we represent the hill as the linear function $f(x) = \frac{2}{3}x$.

But how do we reflect the fact that $f(60) \neq 40$, but instead $f(60) = 0$?

We will use a <u>piecewise function</u>, which uses different formulas for different values of x.

eg Consider a function that behaves like x^2+3 for $x \geq 2$, like $5-x$ for $-1 < x < 2$, and like $\sin(x)$ for $x \leq -1$. Then we could write this function as

$$f(x) = \begin{cases} x^2+3, & \text{for } x \geq 2 \\ 5-x, & \text{for } -1 < x < 2 \\ \sin(x), & \text{for } x \leq -1 \end{cases}$$

An <u>absolute value</u> function is a special kind of piecewise function. This is because

$$|x| = \begin{cases} x, & \text{for } x \geq 0 \\ -x, & \text{for } x < 0 \end{cases}$$

Now we can represent the hill with the pit as the piecewise function

$$f(x) = \begin{cases} \frac{2}{3}x, & \text{for } x \neq 60 \\ 0, & \text{for } x = 60 . \end{cases}$$

As expected, $f(60) = 0$. However, we also have

$$\lim_{x \to 60} f(x) = 40.$$

Thus it is certainly possible that the value of a function and the value of its limit <u>may</u> differ.

However, they can also be equal: here,

$$f(15) = \lim_{x \to 15} f(x) = 10.$$