

## MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

TEST 2

MATHEMATICS 1000-002

OCTOBER 26TH, 2022

Name
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MUN Number
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1. For each of the following, indicate your answer by checking the corresponding box. (You may check only one box per question. If more than one box is checked, you will not receive any credit for that question.)
- [3] (a) Which, if any, of the following is a property of the horizontal asymptotes to a function  $f(x)$ ?
- $f(x)$  may have any number of horizontal asymptotes
  - the graph of  $f(x)$  may never cross a horizontal asymptote
  - a horizontal asymptote describes the manner in which  $f(x)$  becomes unboundedly large (positively or negatively) as  $x$  approaches a real number  $p$
  - a horizontal asymptote describes the behaviour of  $f(x)$  as  $x$  becomes unboundedly large (positively or negatively)
  - none of the above is a property of horizontal asymptotes
- [3] (b) Which, if any, of the following statements would ensure that a function  $f(x)$  has a removable discontinuity at a point  $x = p$ ?
- the definition of  $f(x)$  changes at  $x = p$
  - factors of  $(x - p)$  can be cancelled from both the numerator and denominator of  $f(x)$
  - $f(p)$  is defined, but  $\lim_{x \rightarrow p} f(x)$  does not exist
  - $\lim_{x \rightarrow p} f(x)$  exists, but  $f(p)$  is undefined
  - none of the above would ensure this
- [3] (c) Which, if any, of the following situations is impossible for a function  $f(x)$  at a point  $x = p$ ?
- $f(x)$  is both continuous and differentiable at  $x = p$
  - $f(x)$  is differentiable at  $x = p$  but not continuous at  $x = p$
  - $f(x)$  is continuous at  $x = p$  but not differentiable at  $x = p$
  - $f(x)$  is neither continuous nor differentiable at  $x = p$
  - none of the above is impossible
- [3] (d) Which, if any, of the following does not provide graphical evidence that a function  $f(x)$  is non-differentiable at a point  $x = p$ ?
- $f(x)$  has a horizontal tangent line at  $x = p$
  - $f(x)$  has a vertical tangent line at  $x = p$
  - $f(x)$  has a vertical asymptote at  $x = p$
  - $f(x)$  has an abrupt change or “sharp corner” at  $x = p$
  - none of the above provides this evidence
- [3] (e) Which of the following is neither an example of, nor equivalent to, a rate of change?
- the velocity of an object moving in a straight line
  - the infection rate of a virus
  - the slope of a tangent line to a curve
  - the derivative of a function
  - all of the above are examples of, or are equivalent to, a rate of change

[5] 2. Identify, with justification, any horizontal asymptotes to the graph of  $f(x) = \frac{2x^3(6x-1)}{(3x^2+4)^2}$ .

[8] 3. Given the function

$$f(x) = \begin{cases} x^2 + 5kx, & \text{for } x < 2 \\ 3k^2 - 4, & \text{for } x = 2 \\ k^2x + 4x + 4, & \text{for } x > 2 \end{cases}$$

use the definition of continuity to determine all values of the constant  $k$  for which  $f(x)$  is continuous at  $x = 2$ .

4. Consider the function  $f(x) = \frac{x}{2x + 5}$ .

[8] (a) Use the limit definition of the derivative to find  $f'(x)$ .

[4] (b) Find the equation of the line that is tangent to the curve  $y = \frac{x}{2x + 5}$  at  $x = -3$ .