MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 7 | **MATH 1000** | FALL 2024

SOLUTIONS

[5] 1. (a) First we simplify the function using the properties of logarithms:

$$\begin{aligned} f(x) &= \ln\left(\frac{\sqrt[3]{x}\csc^5(x)}{(3x-1)^7}\right) \\ &= \ln\left(\sqrt[3]{x}\csc^5(x)\right) - \ln\left((3x-1)^7\right) \\ &= \ln\left(\sqrt[3]{x}\right) + \ln\left(\csc^5(x)\right) - \ln\left((3x-1)^7\right) \\ &= \frac{1}{3}\ln(x) + 5\ln(\csc(x)) - 7\ln(3x-1). \end{aligned}$$

Now we differentiate:

$$\begin{aligned} f'(x) &= \frac{1}{3}\cdot\frac{1}{x} + 5\cdot\frac{1}{\csc(x)}\cdot[\csc(x)]' - 7\cdot\frac{1}{3x-1}\cdot[3x-1]' \\ &= \frac{1}{3x} + \frac{5}{\csc(x)}\cdot[-\csc(x)\cot(x)] - \frac{7}{3x-1}\cdot 3 \\ &= \frac{1}{3x} - 5\cot(x) - \frac{21}{3x-1}. \end{aligned}$$

[5] (b) Since this function is of the form $[f(x)]^{g(x)}$, we must use logarithmic differentiation:

$$\begin{aligned} \ln(y) &= \ln\left([\sec(x)]^{x^2-9}\right) \\ &= (x^2-9)\ln(\sec(x)) \\ \frac{d}{dx}[\ln(y)] &= \frac{d}{dx}[(x^2-9)\ln(\sec(x))] \\ \frac{1}{y}\cdot\frac{dy}{dx} &= 2x\ln(\sec(x)) + (x^2-9)\cdot\frac{1}{\sec(x)}\cdot\sec(x)\tan(x) \\ &= 2x\ln(\sec(x)) + (x^2-9)\tan(x) \\ \frac{dy}{dx} &= y\left[2x\ln(\sec(x)) + (x^2-9)\tan(x)\right] \\ &= [\sec(x)]^{x^2-9}\left[2x\ln(\sec(x)) + (x^2-9)\tan(x)\right]. \end{aligned}$$

[3] (c) We use the Chain Rule twice:

$$\begin{aligned}
\frac{dy}{dx} &= -\sin(\cosh(\arccos(x))) \cdot \frac{d}{dx}[\cosh(\arccos(x))] \\
&= -\sin(\cosh(\arccos(x))) \sinh(\arccos(x)) \cdot \frac{d}{dx}[\arccos(x)] \\
&= -\sin(\cosh(\arccos(x))) \sinh(\arccos(x)) \cdot \frac{-1}{\sqrt{1-x^2}} \\
&= \frac{\sin(\cosh(\arccos(x))) \sinh(\arccos(x))}{\sqrt{1-x^2}}.
\end{aligned}$$

[7] 2. First we use the Chain Rule twice (or, alternatively, the Chain Rule followed by the Quotient Rule):

$$\begin{aligned}
y' &= \frac{1}{1+\left(\frac{1}{\ln(x)}\right)^2} \cdot [(\ln(x))^{-1}]' \\
&= \frac{1}{1+\left(\frac{1}{\ln(x)}\right)^2} \cdot -(\ln(x))^{-2} \cdot [\ln(x)]' \\
&= \frac{-1}{[\ln(x)]^2+1} \cdot \frac{1}{x} \\
&= \frac{-1}{x\left(\ln^2(x)+1\right)}.
\end{aligned}$$

The slope of the tangent line is

$$m_T = y'(e) = \frac{-1}{e\left(\ln^2(e)+1\right)} = -\frac{1}{2e}$$

so the slope of the normal line is

$$m_N = -\frac{1}{m_T} = -\frac{1}{-\frac{1}{2e}} = 2e.$$

Also, note that when $x = e$,

$$y = \arctan\left(\frac{1}{\ln(e)}\right) = \arctan(1) = \frac{\pi}{4}.$$

Thus the equation of the tangent line is

$$y - \frac{\pi}{4} = -\frac{1}{2e}(x-e) \quad \Longrightarrow \quad y = -\frac{1}{2e}x + \frac{1}{2} + \frac{\pi}{4}$$

while the equation of the normal line is

$$y - \frac{\pi}{4} = 2e(x-e) \quad \Longrightarrow \quad y = 2ex - 2e^2 + \frac{\pi}{4}.$$