MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 6 **MATHEMATICS 1000** FALL 2024

SOLUTIONS

[4] 1. (a) We use the Product Rule, followed by the Chain Rule (twice):

$$\begin{aligned} g'(t) &= [(t^4+3t)^7]'(5t^2-9)^4 + (t^4+3t)^7[(5t^2-9)^4]' \\ &= 7(t^4+3t)^6[t^4+3t]'(5t^2-9)^4 + (t^4+3t)^7 \cdot 4(5t^2-9)^3[5t^2-9]' \\ &= 7(t^4+3t)^6(4t^3+3)(5t^2-9)^4 + (t^4+3t)^7 \cdot 4(5t^2-9)^3(10t) \\ &= 7(t^4+3t)^6(4t^3+3)(5t^2-9)^4 + 40t(t^4+3t)^7(5t^2-9)^3 \\ &= (t^4+3t)^6(5t^2-9)^3[7(4t^3+3)(5t^2-9) + 40t(t^4+3t)] \\ &= (t^4+3t)^6(5t^2-9)^3(165t^5-252t^3+180t^2-189). \end{aligned}$$

[3] (b) We use the Chain Rule twice:

$$\begin{aligned} y' &= e^{\sqrt{x^2+4}}\left[\sqrt{x^2+4}\right]' \\ &= e^{\sqrt{x^2+4}} \cdot \frac{1}{2}(x^2+4)^{-\frac{1}{2}}[x^2+4]' \\ &= e^{\sqrt{x^2+4}} \cdot \frac{1}{2}(x^2+4)^{-\frac{1}{2}} \cdot 2x \\ &= x(x^2+4)^{-\frac{1}{2}}e^{\sqrt{x^2+4}} \\ &= \frac{xe^{\sqrt{x^2+4}}}{\sqrt{x^2+4}}. \end{aligned}$$

[3] (c) We use the Chain Rule, followed by the Quotient Rule:

$$\begin{aligned} f'(x) &= \cos\left(\frac{5^x}{x^5}\right)\left[\frac{5^x}{x^5}\right]' \\ &= \cos\left(\frac{5^x}{x^5}\right) \cdot \frac{[5^x]'x^5 - 5^x[x^5]'}{(x^5)^2} \\ &= \cos\left(\frac{5^x}{x^5}\right) \cdot \frac{5^x\ln(5)x^5 - 5^x \cdot 5x^4}{x^{10}} \\ &= \cos\left(\frac{5^x}{x^5}\right) \cdot \frac{5^x\ln(5)x - 5^{x+1}}{x^6} \\ &= \frac{\cos\left(\frac{5^x}{x^5}\right)\left[5^x\ln(5)x - 5^{x+1}\right]}{x^6}. \end{aligned}$$

Alternatively, we could first rewrite the given function as

$$f(x) = \sin(5^x x^{-5})$$

and then use the Chain Rule followed by the Product Rule:

$$\begin{aligned} f'(x) &= \cos(5^x x^{-5})[5^x x^{-5}]' \\ &= \cos(5^x x^{-5})([5^x]' x^{-5} + 5^x [x^{-5}]') \\ &= \cos(5^x x^{-5})(5^x \ln(x) x^{-5} + 5^x[-5x^{-6}]) \\ &= \cos(5^x x^{-5})(5^x \ln(x) x^{-5} - 5^{x+1} x^{-6}). \end{aligned}$$

[3] (d) We use the Quotient Rule, followed by the Chain Rule:

$$\begin{aligned} y' &= \frac{[\sec(x^2)]'(x^2+1) - \sec(x^2)[x^2+1]'}{(x^2+1)^2} \\ &= \frac{\sec(x^2)\tan(x^2)[x^2]'(x^2+1) - \sec(x^2)(2x)}{(x^2+1)^2} \\ &= \frac{\sec(x^2)\tan(x^2)(2x)(x^2+1) - \sec(x^2)(2x)}{(x^2+1)^2} \\ &= \frac{2x(x^2+1)\sec(x^2)\tan(x^2) - 2x\sec(x^2)}{(x^2+1)^2}. \end{aligned}$$

[7] 2. First we differentiate both sides of the equation with respect to x:

$$\begin{aligned} [3(x^2+y^2)^2]' &= [100xy]' \\ 6(x^2+y^2)[x^2+y^2]' &= 100y + 100xy' \\ 6(x^2+y^2)(2x+2yy') &= 100y + 100xy' \\ 12x^2yy' + 12y^3y' - 100xy' &= 100y - 12x^3 - 12xy^2 \\ y'[12x^2y + 12y^3 - 100x] &= 100y - 12x^3 - 12xy^2 \\ y' &= \frac{100y - 12x^3 - 12xy^2}{12x^2y + 12y^3 - 100x} \\ &= \frac{25y - 3x^3 - 3xy^2}{3x^2y + 3y^3 - 25x}. \end{aligned}$$

When $x = 1$ and $y = 3$, the slope of the tangent line is

$$y' = \frac{25(3) - 3(1^3) - 3(1)(3^2)}{3(1^2)(3) + 3(3^3) - 25(1)} = \frac{45}{65} = \frac{9}{13}.$$

Thus the equation of the tangent line is given by

$$y - 3 = \frac{9}{13}(x-1) \quad \Longrightarrow \quad y = \frac{9}{13}x + \frac{30}{13}.$$