MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 5 **MATHEMATICS 1000** FALL 2024

SOLUTIONS

[3] 1. (a) We use the Quotient Rule:

$$\begin{aligned} f'(x) &= \frac{[x-\cos(x)]'[x+\cos(x)] - [x-\cos(x)][x+\cos(x)]'}{[x+\cos(x)]^2} \\ &= \frac{[1+\sin(x)][x+\cos(x)] - [x-\cos(x)][1-\sin(x)]}{[x+\cos(x)]^2} \\ &= \frac{2x\sin(x)+2\cos(x)}{[x+\cos(x)]^2}. \end{aligned}$$

[5] (b) We use the Quotient Rule, together with the Product Rule:

$$\begin{aligned} f'(x) &= \frac{[x\cos(x)]'[x+\cos(x)] - x\cos(x)[x+\cos(x)]'}{[x+\cos(x)]^2} \\ &= \frac{([x]'\cos(x) + x[\cos(x)]')[x+\cos(x)] - x\cos(x)[1-\sin(x)]}{[x+\cos(x)]^2} \\ &= \frac{[\cos(x) - x\sin(x)][x+\cos(x)] - x\cos(x)[1-\sin(x)]}{[x+\cos(x)]^2} \\ &= \frac{\cos^2(x) - x^2\sin(x)}{[x+\cos(x)]^2}. \end{aligned}$$

[2] (c) We can first rewrite the function as

$$f(x) = \frac{x}{x\cos(x)} + \frac{\cos(x)}{x\cos(x)} = \frac{1}{\cos(x)} + \frac{1}{x} = \sec(x) + x^{-1}.$$

Then

$$\begin{aligned} f'(x) &= [\sec(x)]' + [x^{-1}]' \\ &= \sec(x)\tan(x) + (-x^{-2}) \\ &= \sec(x)\tan(x) - \frac{1}{x^2}. \end{aligned}$$

[4] (d) We use the Product Rule twice:

$$\begin{aligned} y' &= [e^x]'[\csc(x)\cot(x)] + e^x[\csc(x)\cot(x)]' \\ &= e^x\csc(x)\cot(x) + e^x([\csc(x)]'\cot(x) + \csc(x)[\cot(x)]') \\ &= e^x\csc(x)\cot(x) + e^x([-\csc(x)\cot(x)]\cot(x) + \csc(x)[-\csc^2(x)]) \\ &= e^x\csc(x)\cot(x) - e^x\csc(x)\cot^2(x) - e^x\csc^3(x). \end{aligned}$$

[6] 2. First note that the object is launched from the ground so $s_0 = 0$, and therefore

$$s(t) = \frac{1}{2}gt^2 + v_0t + 0 = -4.9t^2 + v_0t.$$

Its velocity is therefore

$$v(t) = -9.8t + v_0.$$

At the moment when the object reaches its maximum height, its velocity is zero, so at this point in time,

$$-9.8t + v_0 = 0 \quad \Longrightarrow \quad t = \frac{v_0}{9.8}.$$

Therefore

$$\begin{aligned} s\left(\frac{v_0}{9.8}\right) &= -4.9\left(\frac{v_0}{9.8}\right)^2 + v_0\left(\frac{v_0}{9.8}\right) \\ 250 &= -\frac{1}{19.6}v_0^2 + \frac{1}{9.8}v_0^2 \\ &= \frac{1}{19.6}v_0^2 \\ 4900 &= v_0^2 \\ 70 &= v_0. \end{aligned}$$

The initial velocity of the object is 70 m/sec.