MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 5 | **MATHEMATICS 1000** | FALL 2022

SOLUTIONS

[3] 1. (a) We use the Quotient Rule:

$$\begin{aligned} y' &= \frac{[\sqrt{x}+4]'(\sqrt{x}-4) - [\sqrt{x}-4]'(\sqrt{x}+4)}{(\sqrt{x}-4)^2} \\ &= \frac{\frac{1}{2}x^{-\frac{1}{2}}(\sqrt{x}-4) - \frac{1}{2}x^{-\frac{1}{2}}(\sqrt{x}+4)}{(\sqrt{x}-4)^2} \\ &= \frac{\frac{1}{2}\sqrt{x} - 2x^{-\frac{1}{2}} - \frac{1}{2}\sqrt{x} - 2x^{-\frac{1}{2}}}{(\sqrt{x}-4)^2} \\ &= \frac{-4}{\sqrt{x}(\sqrt{x}-4)^2}. \end{aligned}$$

[4] (b) We use the Quotient Rule, followed by the Product Rule:

$$\begin{aligned} y' &= \frac{[x^2e^x]'(5x+1) - [5x+1]'x^2e^x}{(5x+1)^2} \\ &= \frac{[(x^2)'e^x + (e^x)'x^2](5x+1) - 5x^2e^x}{(5x+1)^2} \\ &= \frac{(2xe^x + x^2e^x)(5x+1) - 5x^2e^x}{(5x+1)^2} \\ &= \frac{5x^3e^x + 6x^2e^x + 2xe^x}{(5x+1)^2}. \end{aligned}$$

[4] (c) We use the Product Rule twice:

$$\begin{aligned} f'(t) &= [t^{\frac{4}{3}}]'e^t\cot(t) + [e^t\cot(t)]'t^{\frac{4}{3}} \\ &= \frac{4}{3}t^{\frac{1}{3}}e^t\cot(t) + ([e^t]'\cot(t) + [\cot(t)]'e^t)t^{\frac{4}{3}} \\ &= \frac{4}{3}t^{\frac{1}{3}}e^t\cot(t) + [e^t\cot(t) - e^t\csc^2(t)]t^{\frac{4}{3}} \\ &= \frac{4}{3}t^{\frac{1}{3}}e^t\cot(t) + t^{\frac{4}{3}}e^t\cot(t) - t^{\frac{4}{3}}e^t\csc^2(t). \end{aligned}$$

[5] 2. First we compute

$$\begin{aligned} y' &= [\sin(x)]' \tan(x) + [\tan(x)]' \sin(x) \\ &= \cos(x)\tan(x) + \sec^2(x)\sin(x) \\ &= \sin(x) + \sec^2(x)\sin(x). \end{aligned}$$

At $x = \frac{\pi}{6}$, then, the slope of the tangent line is

$$m_T = \sin\left(\frac{\pi}{6}\right) + \sec^2\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{6}\right) = \frac{1}{2} + \left(\frac{2\sqrt{3}}{3}\right)^2 \cdot \frac{1}{2} = \frac{7}{6}.$$

The y-coordinate when $x = \frac{\pi}{6}$ is

$$y = \sin\left(\frac{\pi}{6}\right)\tan\left(\frac{\pi}{6}\right) = \frac{1}{2} \cdot \frac{\sqrt{3}}{3} = \frac{\sqrt{3}}{6}.$$

Thus the equation of the tangent line is

$$y - \frac{\sqrt{3}}{6} = \frac{7}{6}\left(x - \frac{\pi}{6}\right) \quad \Longrightarrow \quad y = \frac{7}{6}x - \frac{7\pi}{36} + \frac{\sqrt{3}}{6}.$$

Next, the slope of the normal line is

$$m_N = -\frac{1}{m_T} = -\frac{6}{7}.$$

The equation of the normal line, then, is given by

$$y - \frac{\sqrt{3}}{6} = -\frac{6}{7}\left(x - \frac{\pi}{6}\right) \quad \Longrightarrow \quad y = -\frac{6}{7}x + \frac{\pi}{7} + \frac{\sqrt{3}}{6}.$$

[4] 3. By the limit definition of the derivative,

$$\frac{d}{dx}[\cos(x)] = \lim_{h \to 0} \frac{\cos(x+h) - \cos(x)}{h}.$$

The trigonometric identity for the cosine of a sum is

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta),$$

so we can write

$$\begin{aligned} \frac{d}{dx}[\cos(x)] &= \lim_{h \to 0} \frac{\cos(x)\cos(h) - \sin(x)\sin(h) - \cos(x)}{h} \\ &= \lim_{h \to 0} \frac{\cos(x)[\cos(h) - 1] - \sin(x)\sin(h)}{h} \\ &= \lim_{h \to 0} \frac{\cos(x)[\cos(h) - 1]}{h} - \lim_{h \to 0} \frac{\sin(x)\sin(h)}{h} \\ &= \cos(x) \lim_{h \to 0} \frac{\cos(h) - 1}{h} - \sin(x) \lim_{h \to 0} \frac{\sin(h)}{h} \\ &= \cos(x) \cdot 0 - \sin(x) \cdot 1 \\ &= -\sin(x). \end{aligned}$$