

# MEMORIAL UNIVERSITY OF NEWFOUNDLAND

## DEPARTMENT OF MATHEMATICS AND STATISTICS

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SECTION 4.4

Math 1000 Worksheet

FALL 2022

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### SOLUTIONS

1. (a) First we determine the critical points of  $f(x)$ . We have

$$f'(x) = 3x^2 - 9 = 3(x^2 - 3)$$

so  $f'(x) = 0$  when  $x = \pm\sqrt{3}$  and  $f'(x)$  is never undefined. Hence  $x = \pm\sqrt{3}$  are the only critical points. Next we evaluate  $f(x)$  at the critical points and at the endpoints  $x = -4$  and  $x = 3$ :

$$f(-\sqrt{3}) = 6\sqrt{3} \approx 10.4, \quad f(\sqrt{3}) = -6\sqrt{3} \approx -10.4$$

$$f(-4) = -28, \quad f(3) = 0.$$

Hence the maximum value of  $f(x)$  on  $-4 \leq x \leq 3$  is  $6\sqrt{3}$  and the minimum value is  $-28$ .

- (b) First we identify the critical points. Note that

$$f'(x) = \frac{(2x)(x+1) - (1)(x^2+3)}{(x+1)^2} = \frac{(x+3)(x-1)}{(x+1)^2},$$

which is zero for  $x = 1$  and  $x = -3$ , and fails to exist at  $x = -1$ . For the interval  $0 \leq x \leq 4$ , then, the only critical point is  $x = 1$ , for which  $f(1) = 2$ . Checking the endpoints, we have  $f(0) = 3$  and  $f(4) = \frac{19}{5} = 3.8$ . Hence the maximum value of  $f(x)$  is  $\frac{19}{5}$ , and the minimum value is  $2$ .

- (c) We again begin by determining the critical points of  $f(x)$ , observing that

$$f'(x) = \sec(x) \tan(x).$$

Setting  $f'(x) = 0$  gives  $x = 0$ , and  $f'(x)$  never fails to exist on the given interval (though it does fail to exist for many other values of  $x$ ). Hence  $x = 0$  is the only critical point. We evaluate  $f(x)$  there and at the endpoints, giving

$$f(0) = 1, \quad f\left(-\frac{\pi}{6}\right) = \frac{2\sqrt{3}}{3} \approx 1.15, \quad f\left(\frac{\pi}{3}\right) = 2.$$

Thus the maximum value of  $f(x)$  on the given interval is  $2$  and the minimum value is  $1$ .

(d) As before, we find the critical points. Differentiation gives

$$f'(x) = 1 + 2 \sin(x)$$

which fails to exist nowhere, and is zero for  $x = -\frac{\pi}{6}$  and  $x = -\frac{5\pi}{6}$ , both of which are on the interval  $-\pi \leq x \leq \pi$ . Note that

$$f\left(-\frac{\pi}{6}\right) = -\frac{\pi}{6} - 2 \cos\left(-\frac{\pi}{6}\right) \approx -2.26$$

and

$$f\left(-\frac{5\pi}{6}\right) = -\frac{5\pi}{6} - 2 \cos\left(-\frac{5\pi}{6}\right) \approx -0.89.$$

At the endpoints,

$$f(-\pi) = -\pi - 2 \cos(-\pi) \approx -1.14 \quad \text{and} \quad f(\pi) = \pi - 2 \cos(\pi) \approx 5.14.$$

Hence the maximum value of  $f(x)$  on  $-\pi \leq x \leq \pi$  is approximately **5.14**, while the minimum value is about **-2.26**.

2. (a) Observe that

$$f'(x) = 12x - 6x^2 = 6x(2 - x).$$

This is never undefined, and equals zero for  $x = 0$  and  $x = 2$ , of which only the latter lies on the indicated interval. Since the requirements of the Second Derivative Test are satisfied, we find

$$f''(x) = 12 - 12x \quad \implies \quad f''(2) = -12 < 0.$$

Thus  $x = 2$  is the absolute maximum, and the maximum value of  $f(x)$  on the interval  $1 < x < 7$  is  **$f(2) = 10$** .

(b) From part (a), we know that  $x = 0$  is the only critical point on the interval  $-7 < x < 1$ , and  $f''(0) = 12 > 0$ . Thus  $x = 0$  is the absolute minimum, and the minimum value of  $f(x)$  on this interval is  **$f(0) = 2$** .

3. We have

$$f'(x) = \frac{12 + 16x - 3x^2}{(3x - 8)^2}.$$

This is undefined for  $x = \frac{8}{3}$ , which is not on the given interval. It is zero when  $x = 6$  and  $x = -\frac{2}{3}$ , of which only the latter is on  $-2 < x < 2$ . Since we have only one critical point, we can use the Second Derivative Test. Observe that

$$f''(x) = -\frac{200}{(3x - 8)^3} \quad \implies \quad f''\left(-\frac{2}{3}\right) = \frac{1}{5} > 0.$$

Thus  $x = -\frac{2}{3}$  is the absolute minimum, and the minimum value of  $f(x)$  on this interval is  **$f\left(-\frac{2}{3}\right) = \frac{1}{5}$** .