

MEMORIAL UNIVERSITY OF NEWFOUNDLAND
DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 3.7

Math 1000 Worksheet

FALL 2024

SOLUTIONS

1. We compute each derivative in turn:

$$\begin{aligned}
 f'(x) &= (x^2)'e^x + (e^x)'x^2 = 2xe^x + x^2e^x \\
 f''(x) &= [(2x)'e^x + (e^x)'(2x)] + [(x^2)'e^x + (e^x)'x^2] \\
 &= [2e^x + 2xe^x] + [2xe^x + x^2e^x] = 2e^x + 4xe^x + x^2e^x \\
 f'''(x) &= 2(e^x)' + [(4x)'e^x + (e^x)'(4x)] + [(x^2)'e^x + (e^x)'x^2] \\
 &= 2e^x + [4e^x + 4xe^x] + [2xe^x + x^2e^x] \\
 &= 6e^x + 6xe^x + x^2e^x.
 \end{aligned}$$

2. We have

$$\begin{aligned}
 f'(x) &= (x)' \sin(x) + [\sin(x)]'x = (1) \sin(x) + \cos(x)x = \sin(x) + x \cos(x) \\
 f''(x) &= [\sin(x)]' + (x)' \cos(x) + [\cos(x)]'x = \cos(x) + (1) \cos(x) + [-\sin(x)]x \\
 &= 2 \cos(x) - x \sin(x) \\
 f'''(x) &= [2 \cos(x)]' - (x)' \sin(x) - [\sin(x)]'x = -2 \sin(x) - (1) \sin(x) - \cos(x)x \\
 &= -3 \sin(x) - x \cos(x) \\
 f^{(4)}(x) &= [-3 \sin(x)]' - (x)' \cos(x) - x[\cos(x)]' \\
 &= -3 \cos(x) - (1) \cos(x) - x[-\sin(x)] = -4 \cos(x) + x \sin(x).
 \end{aligned}$$

3. We use the Chain Rule to obtain the first derivative:

$$y' = \sec^2(x^2) \cdot (x^2)' = 2x \sec^2(x^2).$$

Next we need to use both the Chain Rule and the Product Rule to find the second derivative:

$$\begin{aligned}
 y'' &= (2x)' \sec^2(x^2) + [\sec^2(x^2)]'(2x) \\
 &= 2 \sec^2(x^2) + 2 \sec(x^2) \cdot [\sec(x^2)]'(2x) \\
 &= 2 \sec^2(x^2) + 2 \sec(x^2) \cdot \sec(x^2) \tan(x^2) \cdot (x^2)'(2x) \\
 &= 2 \sec^2(x^2) + 2 \sec(x^2) \cdot \sec(x^2) \tan(x^2) \cdot (2x)(2x) \\
 &= 2 \sec^2(x^2) + 8x^2 \sec^2(x^2) \tan(x^2).
 \end{aligned}$$

4. We begin by finding the first derivative:

$$\begin{aligned}
 \frac{d}{dx}[y] &= \frac{d}{dx}[x^2 + \sin(y)] \\
 \frac{dy}{dx} &= 2x + \cos(y) \frac{dy}{dx} \\
 \frac{dy}{dx} - \cos(y) \frac{dy}{dx} &= 2x \\
 \frac{dy}{dx}[1 - \cos(y)] &= 2x \\
 \frac{dy}{dx} &= \frac{2x}{1 - \cos(y)}.
 \end{aligned}$$

We differentiate again, substituting the expression for $\frac{dy}{dx}$ where possible:

$$\begin{aligned}
 \frac{d^2y}{dx^2} &= \frac{d}{dx} \left[\frac{2x}{1 - \cos(y)} \right] \\
 &= \frac{2[1 - \cos(y)] - 2x \sin(y) \frac{dy}{dx}}{[1 - \cos(y)]^2} \\
 &= \frac{2 - 2\cos(y) - 2x \sin(y) \cdot \frac{2x}{1 - \cos(y)}}{[1 - \cos(y)]^2} \\
 &= \frac{2[1 - \cos(y)]^2 - 4x^2 \sin(y)}{[1 - \cos(y)]^3} \\
 &= \frac{2 - 4\cos(y) + 2\cos^2(y) - 4x^2 \sin(y)}{[1 - \cos(y)]^3}.
 \end{aligned}$$

5. We differentiate implicitly to find the first derivative:

$$\begin{aligned}
 \frac{d}{dx} [\sqrt{x} + \sqrt{y}] &= \frac{d}{dx}[2] \\
 \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}y^{-\frac{1}{2}} \frac{dy}{dx} &= 0 \\
 \frac{dy}{dx} &= -\left(\frac{y}{x}\right)^{\frac{1}{2}}.
 \end{aligned}$$

Differentiating implicitly a second time gives us

$$\begin{aligned}
 \frac{d^2y}{dx^2} &= -\frac{1}{2} \left(\frac{y}{x}\right)^{-\frac{1}{2}} \left(\frac{\frac{d}{dx}[y]x - \frac{d}{dx}[x]y}{x^2} \right) = -\frac{1}{2} \left(\frac{y}{x}\right)^{-\frac{1}{2}} \left(\frac{\left(\frac{dy}{dx}\right)x - (1)y}{x^2} \right) \\
 &= -\frac{1}{2} \left(\frac{x}{y}\right)^{\frac{1}{2}} \left(\frac{-\left(\frac{y}{x}\right)^{\frac{1}{2}}x - y}{x^2} \right) = \frac{\sqrt{x} + \sqrt{y}}{2x^{\frac{3}{2}}}.
 \end{aligned}$$

But remember that $\sqrt{x} + \sqrt{y} = 2$, so this becomes

$$\frac{d^2y}{dx^2} = \frac{2}{2x^{\frac{3}{2}}} = x^{-\frac{3}{2}}.$$

6. We have

$$\begin{aligned} v(t) &= s'(t) = \frac{(49t - 10)'(t + 10) - (t + 10)'(49t - 10)}{(t + 10)^2} \\ &= \frac{(49)(t + 10) - (1)(49t - 10)}{(t + 10)^2} = \frac{500}{(t + 10)^2} = \frac{500}{t^2 + 20t + 100} \\ a(t) &= s''(t) = \frac{(500)'(t^2 + 20t + 100) - (t^2 + 20t + 100)'(500)}{(t^2 + 20t + 100)^2} \\ &= \frac{0 - (2t + 20)(500)}{(t^2 + 20t + 100)^2} = \frac{-1000(t + 10)}{(t + 10)^4} = -\frac{1000}{(t + 10)^3}. \end{aligned}$$

Therefore,

$$v(0) = \frac{500}{100} = 5 \quad \text{and} \quad a(0) = \frac{-1000}{1000} = -1.$$

Hence the puppy's initial velocity is 5 cm/sec and its initial acceleration is -1 cm/sec².