MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 3.6 | **Math 1000 Worksheet** | FALL 2024

SOLUTIONS

1. (a) $\dfrac{dy}{dx} = \cosh(x^3) \cdot \dfrac{d}{dx}[x^3] = 3x^2 \cosh(x^3)$

   (b) $\dfrac{dy}{dx} = 3\sinh^2(x) \cdot \dfrac{d}{dx}[\sinh(x)] = 3\sinh^2(x)\cosh(x)$

   (c) $f'(x) = \dfrac{[\cosh(x)]'\cos(x) - [\cos(x)]'\cosh(x)}{[\cos(x)]^2} = \dfrac{\sinh(x)\cos(x) - [-\sin(x)]\cosh(x)}{\cos^2(x)}$

   $= \dfrac{\sinh(x)\cos(x) + \sin(x)\cosh(x)}{\cos^2(x)}$

   (d) We must use logarithmic differentiation:

$$\ln(y) = \ln(x^{\cosh(x)}) = \cosh(x)\ln(x)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \sinh(x)\ln(x) + \cosh(x) \cdot \frac{1}{x}$$

$$\frac{dy}{dx} = y\left[\sinh(x)\ln(x) + \frac{\cosh(x)}{x}\right] = x^{\cosh(x)}\left[\sinh(x)\ln(x) + \frac{\cosh(x)}{x}\right].$$

2. Since $\sinh(x) = \dfrac{e^x - e^{-x}}{2}$,

$$\frac{d}{dx}[\sinh(x)] = \frac{d}{dx}\left[\frac{e^x - e^{-x}}{2}\right] = \frac{1}{2}[e^x - e^{-x} \cdot (-1)] = \frac{e^x + e^{-x}}{2} = \cosh(x).$$

3. We start from the righthand side:

$$\sinh(x)\cosh(y) + \cosh(x)\sinh(y)$$

$$= \left(\frac{e^x - e^{-x}}{2}\right)\left(\frac{e^y + e^{-y}}{2}\right) + \left(\frac{e^x + e^{-x}}{2}\right)\left(\frac{e^y - e^{-y}}{2}\right)$$

$$= \frac{e^x e^y - e^{-x}e^y + e^x e^{-y} - e^{-x}e^{-y}}{4} + \frac{e^x e^y + e^{-x}e^y - e^x e^{-y} - e^{-x}e^{-y}}{4}$$

$$= \frac{2e^x e^y - 2e^{-x}e^{-y}}{4} = \frac{e^{x+y} - e^{-(x+y)}}{2} = \sinh(x+y).$$