

MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 3.6

Math 1000 Worksheet

FALL 2022

SOLUTIONS

1. (a) $\frac{dy}{dx} = \cosh(x^3) \cdot \frac{d}{dx}[x^3] = 3x^2 \cosh(x^3)$

(b) $\frac{dy}{dx} = 3 \sinh^2(x) \cdot \frac{d}{dx}[\sinh(x)] = 3 \sinh^2(x) \cosh(x)$

(c)
$$f'(x) = \frac{[\operatorname{sech}(x)]' \sec(x) - [\sec(x)]' \operatorname{sech}(x)}{[\sec(x)]^2}$$

$$= \frac{-\operatorname{sech}(x) \tanh(x) \sec(x) - \sec(x) \tan(x) \operatorname{sech}(x)}{\sec^2(x)}$$

$$= -\frac{\operatorname{sech}(x) \tanh(x) + \tan(x) \operatorname{sech}(x)}{\sec(x)}$$

(d) We must use logarithmic differentiation:

$$\ln(y) = \ln(x^{\tanh(x)}) = \tanh(x) \ln(x)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \operatorname{sech}^2(x) \ln(x) + \tanh(x) \cdot \frac{1}{x}$$

$$\frac{dy}{dx} = y \left[\operatorname{sech}^2(x) \ln(x) + \frac{\tanh(x)}{x} \right] = x^{\tanh(x)} \left[\operatorname{sech}^2(x) \ln(x) + \frac{\tanh(x)}{x} \right].$$

2. Since $\sinh(x) = \frac{e^x - e^{-x}}{2}$,

$$\frac{d}{dx}[\sinh(x)] = \frac{d}{dx} \left[\frac{e^x - e^{-x}}{2} \right] = \frac{1}{2} [e^x - e^{-x} \cdot (-1)] = \frac{e^x + e^{-x}}{2} = \cosh(x).$$

3. We start from the righthand side:

$$\begin{aligned} & \sinh(x) \cosh(y) + \cosh(x) \sinh(y) \\ &= \left(\frac{e^x - e^{-x}}{2} \right) \left(\frac{e^y + e^{-y}}{2} \right) + \left(\frac{e^x + e^{-x}}{2} \right) \left(\frac{e^y - e^{-y}}{2} \right) \\ &= \frac{e^x e^y - e^{-x} e^y + e^x e^{-y} - e^{-x} e^{-y}}{4} + \frac{e^x e^y + e^{-x} e^y - e^x e^{-y} - e^{-x} e^{-y}}{4} \\ &= \frac{2e^x e^y - 2e^{-x} e^{-y}}{4} = \frac{e^{x+y} - e^{-(x+y)}}{2} = \sinh(x+y). \end{aligned}$$