MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 3.4 | **Math 1000 Worksheet** | FALL 2022

SOLUTIONS

1. (a) $f'(x) = 2^{\ln(x)}\ln(2)\cdot[\ln(x)]' = \dfrac{2^{\ln(x)}\ln(2)}{x}$

   (b) Rewrite: $y = \log_3(7) + 5\log_3(x)$

   Differentiate: $\dfrac{dy}{dx} = 0 + 5\cdot\dfrac{1}{x\ln(3)} = \dfrac{5}{x\ln(3)}$

   (c) Rewrite: $y = \ln(5) - 7\ln(x)$

   Differentiate: $\dfrac{dy}{dx} = 0 - 7\cdot\dfrac{1}{x} = -\dfrac{7}{x}$

   (d) Rewrite: $g(x) = \ln(x^2-6) - \ln(x^2+4)$

   Differentiate:
$$\begin{aligned} g'(x) &= \frac{1}{x^2-6}\cdot(x^2-6)' - \frac{1}{x^2+4}\cdot(x^2+4)' \\ &= \frac{1}{x^2-6}\cdot(2x) - \frac{1}{x^2+4}\cdot(2x) = \frac{2x}{x^2-6} - \frac{2x}{x^2+4} \\ &= \frac{2x(x^2+4)-2x(x^2-6)}{(x^2-6)(x^2+4)} = \frac{20x}{(x^2-6)(x^2+4)} \end{aligned}$$

2. (a) We take logarithms of both sides and then differentiate:

$$\begin{aligned} \ln(y) &= \ln\left[\frac{x^{3x}(2-x)^4}{(5x^5+1)\sqrt[3]{\csc^2(x)}}\right] \\ &= 3x\ln(x) + 4\ln(2-x) - \ln(5x^5+1) - \frac{2}{3}\ln(\csc(x)) \\ \frac{1}{y}\frac{dy}{dx} &= \frac{d}{dx}[3x]\ln(x) + \frac{d}{dx}[\ln(x)](3x) + 4\cdot\frac{1}{2-x}\cdot\frac{d}{dx}[2-x] \\ &\quad - \frac{1}{5x^5+1}\cdot\frac{d}{dx}[5x^5+1] - \frac{2}{3}\cdot\frac{1}{\csc(x)}\cdot\frac{d}{dx}[\csc(x)] \\ &= 3\ln(x) + \frac{1}{x}\cdot(3x) + \frac{4}{2-x}\cdot(-1) - \frac{1}{5x^5+1}\cdot(25x^4) \\ &\quad - \frac{2}{3\csc(x)}\cdot[-\csc(x)\cot(x)] \\ &= 3\ln(x) + 3 - \frac{4}{2-x} - \frac{25x^4}{5x^5+1} + \frac{2}{3}\cot(x) \\ \frac{dy}{dx} &= \frac{x^{3x}(2-x)^4}{(5x^5+1)\sqrt[3]{\csc^2(x)}}\left[3\ln(x) + 3 - \frac{4}{2-x} - \frac{25x^4}{5x^5+1} + \frac{2}{3}\cot(x)\right]. \end{aligned}$$

(b) We first take logarithms of both sides:

$$\begin{aligned}\ln(y) &= \ln\left[\frac{\sqrt{x^2+1}}{(3x+2)[\tan(x)]^x}\right] \\ &= \frac{1}{2}\ln(x^2+1) - \ln(3x+2) - x\ln(\tan(x)).\end{aligned}$$

Then we differentiate:

$$\begin{aligned}\frac{1}{y}\frac{dy}{dx} &= \frac{1}{2}\cdot\frac{1}{x^2+1}\cdot\frac{d}{dx}[x^2+1] - \frac{1}{3x+2}\cdot\frac{d}{dx}[3x+2] - \frac{d}{dx}[x]\ln(\tan(x)) \\ &\quad - \frac{d}{dx}[\ln(\tan(x))]x \\ &= \frac{1}{2}\cdot\frac{1}{x^2+1}\cdot(2x) - \frac{1}{3x+2}\cdot(3) - (1)\ln(\tan(x)) - \frac{1}{\tan(x)}\cdot\frac{d}{dx}[\tan(x)]x \\ &= \frac{x}{x^2+1} - \frac{3}{3x+2} - \ln(\tan(x)) - x\csc(x)\sec(x) \\ \frac{dy}{dx} &= \frac{\sqrt{x^2+1}}{(3x+2)[\tan(x)]^x}\left[\frac{x}{x^2+1} - \frac{3}{3x+2} - \ln(\tan(x)) - x\csc(x)\sec(x)\right].\end{aligned}$$