MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 3.1 **Math 1000 Worksheet** FALL 2022

SOLUTIONS

1. (a) Rewrite: $f(x) = e^2 e^x$

   Differentiate: $f'(x) = e^2 \dfrac{d}{dx}[e^x] = e^2 e^x = e^{x+2}$

   (b) Rewrite: $g(x) = 5\sin(x) - \dfrac{1}{2}x^{\frac{1}{2}}$

   Differentiate: $g'(x) = 5\cos(x) - \dfrac{1}{2}\left(\dfrac{1}{2}x^{-\frac{1}{2}}\right) = 5\cos(x) - \dfrac{1}{4\sqrt{x}}$

   (c) $f'(t) = \dfrac{7}{3}t^{\frac{4}{3}} - [-\sin(t)] + 0 = \dfrac{7}{3}t^{\frac{4}{3}} + \sin(t)$

   (d) $\dfrac{dy}{dx} = (2x^4)'\tan(x) + [\tan(x)]'(2x^4) = 8x^3\tan(x) + 2x^4\sec^2(x)$

   (e) $g'(\theta) = [\sin(\theta)]'\tan(\theta) + [\tan(\theta)]'\sin(\theta) = \cos(\theta)\tan(\theta) + \sec^2(\theta)\sin(\theta)$

   (f) Rewrite: $f(t) = t^{-1}\csc(t)$

   Differentiate:
   $$\begin{aligned} f'(t) &= (t^{-1})'\csc(t) + [\csc(t)]'t^{-1} \\ &= -t^{-2}\csc(t) - t^{-1}\csc(t)\cot(t) \\ &= -\frac{\csc(t) + t\csc(t)\cot(t)}{t^2} \end{aligned}$$

   (g)
   $$\begin{aligned} f'(x) &= \frac{[1-\sec(x)]'[1+\sec(x)] - [1+\sec(x)]'[1-\sec(x)]}{[1+\sec(x)]^2} \\ &= \frac{-\sec(x)\tan(x)[1+\sec(x)] - \sec(x)\tan(x)[1-\sec(x)]}{[1+\sec(x)]^2} \\ &= \frac{-2\sec(x)\tan(x)}{[1+\sec(x)]^2} \end{aligned}$$

   (h) $\dfrac{dy}{dx} = (x^3e^x)'\cot(x) + [\cot(x)]'x^3e^x = [(x^3)'e^x + (e^x)'x^3]\cot(x) - \csc^2(x)x^3e^x$

   $= (3x^2e^x + x^3e^x)\cot(x) - x^3e^x\csc^2(x)$

   (i) First we use the Quotient Rule:

   $$f'(x) = \frac{(xe^x)'(\sqrt{x}-3) - (\sqrt{x}-3)'(xe^x)}{(\sqrt{x}-3)^2}.$$

   Next we use the Product Rule to evaluate the first derivative in the numerator:

   $$f'(x) = \frac{[(x)'e^x + (e^x)'x](\sqrt{x}-3) - \frac{1}{2}x^{-\frac{1}{2}}xe^x}{(\sqrt{x}-3)^2} = \frac{(e^x + xe^x)(\sqrt{x}-3) - \frac{1}{2}x^{-\frac{1}{2}}xe^x}{(\sqrt{x}-3)^2}.$$

2. Differentiating, we have

$$f'(x) = 2\sec^2(x) - \sqrt{2}\cos(x)$$

$$m_T = f'\left(\frac{\pi}{4}\right) = 2\sec^2\left(\frac{\pi}{4}\right) - \sqrt{2}\cos\left(\frac{\pi}{4}\right) = 2\left(\sqrt{2}\right)^2 - \sqrt{2}\left(\frac{\sqrt{2}}{2}\right) = 3.$$

So then, using point-slope form, the equation of the tangent line is

$$y - 1 = 3\left(x - \frac{\pi}{4}\right) \implies y = 3x - \frac{3\pi}{4} + 1.$$

The slope of the normal line must be

$$m_N = -\frac{1}{3}$$

and hence its equation is

$$y - 1 = -\frac{1}{3}\left(x - \frac{\pi}{4}\right) \implies y = -\frac{1}{3}x + \frac{\pi}{12} + 1.$$

3. Much like the proof that $\frac{d}{dx}[\sec(x)] = \sec(x)\tan(x)$, we write

$$\begin{aligned}\frac{d}{dx}[\csc(x)] &= \frac{d}{dx}\left[\frac{1}{\sin(x)}\right] = \frac{\frac{d}{dx}[1]\sin(x) - \frac{d}{dx}[\sin(x)](1)}{\sin^2(x)} \\ &= \frac{(0)\sin(x) - [\cos(x)](1)}{\sin^2(x)} = -\frac{\cos(x)}{\sin^2(x)} = -\frac{1}{\sin(x)}\cdot\frac{\cos(x)}{\sin(x)} \\ &= -\csc(x)\cot(x).\end{aligned}$$

4. Following the proof that $\frac{d}{dx}[\cot(x)] = -\csc^2(x)$, we have

$$\begin{aligned}\frac{d}{dx}[\tan(x)] &= \frac{d}{dx}\left[\frac{\sin(x)}{\cos(x)}\right] = \frac{\frac{d}{dx}[\sin(x)]\cos(x) - \frac{d}{dx}[\cos(x)]\sin(x)}{\cos^2(x)} \\ &= \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)} = \sec^2(x).\end{aligned}$$