MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

Assignment 2 **MATHEMATICS 1000** Fall 2022

SOLUTIONS

[4] 1. (a) Direct substitution results in a $\frac{0}{0}$ indeterminate form, so we use the rationalisation method:

$$\begin{aligned}
\lim_{h\to 3} \frac{6+h-h^2}{\sqrt{5h-6}-\sqrt{h+6}} &= \lim_{h\to 3} \frac{6+h-h^2}{\sqrt{5h-6}-\sqrt{h+6}} \cdot \frac{\sqrt{5h-6}+\sqrt{h+6}}{\sqrt{5h-6}+\sqrt{h+6}} \\
&= \lim_{h\to 3} \frac{(6+h-h^2)\left(\sqrt{5h-6}+\sqrt{h+6}\right)}{(5h-6)-(h+6)} \\
&= \lim_{h\to 3} \frac{(6+h-h^2)\left(\sqrt{5h-6}+\sqrt{h+6}\right)}{4h-12} \\
&= \lim_{h\to 3} \frac{-(h-3)(h+2)\left(\sqrt{5h-6}+\sqrt{h+6}\right)}{4(h-3)} \\
&= \lim_{h\to 3} \frac{-(h+2)\left(\sqrt{5h-6}+\sqrt{h+6}\right)}{4} \\
&= \frac{-5(3+3)}{4} \\
&= -\frac{15}{2}.
\end{aligned}$$

[4] (b) We can write

$$\begin{aligned}
7(x^2+3x+2)^{-1}-(x+8)(x+1)^{-1} &= \frac{7}{x^2+3x+2}-\frac{x+8}{x+1} \\
&= \frac{7}{(x+2)(x+1)}-\frac{x+8}{x+1} \\
&= \frac{7}{(x+2)(x+1)}-\frac{(x+8)(x+2)}{(x+2)(x+1)} \\
&= \frac{7-(x^2+10x+16)}{(x+2)(x+1)} \\
&= \frac{-x^2-10x-9}{(x+2)(x+1)}.
\end{aligned}$$

Now direct substitution results in a $\frac{0}{0}$ indeterminate form, so we can use the cancellation method:

$$\begin{aligned}\lim_{x\to-1}[7(x^2+3x+2)^{-1}-(x+8)(x+1)^{-1}] &= \lim_{x\to-1}\frac{-x^2-10x-9}{(x+2)(x+1)}\\ &= \lim_{x\to-1}-\frac{(x+9)(x+1)}{(x+2)(x+1)}\\ &= \lim_{x\to-1}-\frac{x+9}{x+2}\\ &= -8.\end{aligned}$$

[4] (c) First observe that

$$\frac{4}{t\cot(7t)} = \frac{4}{t\cdot\frac{\cos(t)}{\sin(t)}} = \frac{4\sin(7t)}{t\cos(t)}.$$

Now we can use the special sine limit. Because the argument of the sine function is $7t$, and there is already a factor of t in the denominator, we need to multiply the numerator and the denominator by 7, to get

$$\begin{aligned}\lim_{t\to0}\frac{4}{t\cot(7t)} &= \lim_{t\to0}\frac{4\sin(7t)}{t\cos(7t)}\cdot\frac{7}{7}\\ &= \lim_{t\to0}\frac{\sin(7t)}{7t}\cdot\frac{28}{\cos(7t)}\\ &= \lim_{t\to0}\frac{\sin(7t)}{7t}\cdot\lim_{t\to0}\frac{28}{\cos(7t)}.\end{aligned}$$

As $t\to0$, it is also true that $7t\to0$ so the first limit is the special sine limit, and the other limit can be evaluated by direct substitution:

$$\lim_{t\to0}\frac{4}{t\cot(7t)} = 1\cdot\lim_{t\to0}\frac{28}{\cos(7t)} = 1\cdot\frac{28}{\cos(0)} = 28.$$

[8] 2. First we set

$$\begin{aligned}x^5-4x^3&=0\\ x^3(x^2-4)&=0\\ x^3(x-2)(x+2)&=0,\end{aligned}$$

so the possible vertical asymptotes are $x=0$, $x=2$ and $x=-2$.

At $x=0$, the numerator is 0 as well, so we need to check the limit:

$$\lim_{x\to0}f(x) = \lim_{x\to0}\frac{x(x+2)}{x^3(x-2)(x+2)} = \lim_{x\to0}\frac{1}{x^2(x-2)}.$$

Now direct substitution results in a $\frac{1}{0}$ form, so $x = 0$ is a vertical asymptote.

At $x = 2$, the numerator is 8, so $x = 2$ is also a vertical asymptote.

At $x = -2$, the numerator is 0, so again we need to determine the limit:

$$\lim_{x \to -2} f(x) = \lim_{x \to -2} \frac{x(x+2)}{x^3(x-2)(x+2)} = \lim_{x \to -2} \frac{1}{x^2(x-2)} = \frac{1}{4(-4)} = -\frac{1}{16}.$$

Since the limit exists, $x = -2$ is not a vertical asymptote.

Now consider the expression $\frac{1}{x^2(x-2)}$. As $x \to 0^-$, the denominator becomes a small negative number, so

$$\lim_{x \to 0^-} f(x) = -\infty.$$

The same is true as $x \to 0^+$, so

$$\lim_{x \to 0^+} f(x) = -\infty.$$

As $x \to 2^-$, the denominator is also a small negative number, so again

$$\lim_{x \to 2^-} f(x) = -\infty.$$

However, as $x \to 2^+$, the denominator is a small positive number, so

$$\lim_{x \to 2^+} f(x) = \infty.$$