MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

Section 2.4 | **Math 1000 Worksheet** | Fall 2022

SOLUTIONS

1. (a) $$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}[x^3-1](x^4+3x^2-x)+\frac{d}{dx}[x^4+3x^2-x](x^3-1)\\ &= (3x^2-0)(x^4+3x^2-x)+(4x^3+6x-1)(x^3-1)\\ &= 7x^6+15x^4-8x^3-6x+1\end{aligned}$$

(b) $$\begin{aligned}f'(x) &= (x+1)'(2x+1)(3x+1)+[(2x+1)(3x+1)]'(x+1)\\ &= (1+0)(2x+1)(3x+1)+[(2x+1)'(3x+1)+(3x+1)'(2x+1)](x+1)\\ &= (2x+1)(3x+1)+[(2+0)(3x+1)+(3+0)(2x+1)](x+1)\\ &= 18x^2+22x+6\end{aligned}$$

(c) $$\begin{aligned}g'(x) &= \frac{(ax^2+b)'(cx^2-d)-(cx^2-d)'(ax^2+b)}{(cx^2-d)^2}\\ &= \frac{2ax(cx^2-d)-2cx(ax^2+b)}{(cx^2-d)^2} = \frac{-2(ad+bc)x}{(cx^2-d)^2}\end{aligned}$$

(d) Rewrite: $f(x) = \dfrac{1}{\frac{x^3+4x^2+4}{x+4}} = \dfrac{x+4}{x^3+4x^2+4}$

Differentiate: $$\begin{aligned}f'(x) &= \frac{(x+4)'(x^3+4x^2+4)-(x^3+4x^2+4)'(x+4)}{(x^3+4x^2+4)^2}\\ &= \frac{(x^3+4x^2+4)-(3x^2+8x)(x+4)}{(x^3+4x^2+4)^2}\\ &= \frac{-2x^3-16x^2-32x+4}{(x^3+4x^2+4)^2}\end{aligned}$$

2. Let $A(x) = \dfrac{f(x)}{g(x)}$ so

$$\begin{aligned}\frac{d}{dx}[A(x)] &= \lim_{h\to 0}\frac{A(x+h)-A(x)}{h} = \lim_{h\to 0}\frac{\frac{f(x+h)}{g(x+h)}-\frac{f(x)}{g(x)}}{h}\\ &= \lim_{h\to 0}\frac{f(x+h)g(x)-f(x)g(x+h)}{hg(x)g(x+h)}\\ &= \lim_{h\to 0}\frac{1}{g(x)g(x+h)}\cdot\lim_{h\to 0}\frac{f(x+h)g(x)-f(x)g(x+h)}{h}\\ &= \frac{1}{[g(x)]^2}\cdot\lim_{h\to 0}\frac{f(x+h)g(x)-f(x)g(x+h)}{h}.\end{aligned}$$

Much like the proof of the Product Rule, we now subtract and add $f(x)g(x)$ to the numerator in the limit:

$$\begin{aligned}\frac{d}{dx}[A(x)] &= \frac{1}{[g(x)]^2}\lim_{h\to 0}\frac{f(x+h)g(x)-f(x)g(x)+f(x)g(x)-f(x)g(x+h)}{h}\\ &= \frac{1}{[g(x)]^2}\cdot\left[\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\cdot g(x)-\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}\cdot f(x)\right]\\ &= \frac{\frac{d}{dx}[f(x)]g(x)-\frac{d}{dx}[g(x)]f(x)}{[g(x)]^2}.\end{aligned}$$