

MEMORIAL UNIVERSITY OF NEWFOUNDLAND
DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 1.3

Math 1000 Worksheet

FALL 2022

SOLUTIONS

1. (a) Using Basic Limit Property #2,

$$\lim_{x \rightarrow p} [f(x) - g(x)] = \lim_{x \rightarrow p} f(x) - \lim_{x \rightarrow p} g(x) = -5 - 4 = -9.$$

- (b) Using Basic Limit Property #s 2 and 3,

$$\lim_{x \rightarrow p} [g(x) - 2f(x)] = \lim_{x \rightarrow p} g(x) - 2 \lim_{x \rightarrow p} f(x) = 4 - 2(-5) = 14.$$

- (c) Using Basic Limit Property #5,

$$\lim_{x \rightarrow p} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow p} f(x)}{\lim_{x \rightarrow p} g(x)} = \frac{-5}{4} = -\frac{5}{4}.$$

- (d) Using Basic Limit Property #4 and the property for radicals of functions,

$$\lim_{x \rightarrow p} f(x) \sqrt{g(x)} = \lim_{x \rightarrow p} f(x) \cdot \sqrt{\lim_{x \rightarrow p} g(x)} = -5 \cdot \sqrt{4} = -10.$$

2. (a) By direct substitution, $\lim_{x \rightarrow 5} (x^2 - 9x - 3) = 5^2 - 9(5) + 3 = 25 - 45 + 3 = -17.$

(b) By direct substitution, $\lim_{x \rightarrow -3} \frac{\sqrt{1-x}}{x} = \frac{\sqrt{4}}{-3} = -\frac{2}{3}.$

(c) By direct substitution, $\lim_{h \rightarrow 0} \frac{\cos(h)}{2^h} = \frac{\cos(0)}{2^0} = 1.$

- (d) Observe that

$$|x - 2| = \begin{cases} x - 2 & \text{for } x \geq 2 \\ -(x - 2) & \text{for } x < 2. \end{cases}$$

Since this is a piecewise function which changes definition at $x = 2$, we must evaluate both the lefthand and righthand limits as $x \rightarrow 2$.

For the lefthand limit (where $x < 2$), we can write $|x - 2| = -(x - 2)$, giving

$$\lim_{x \rightarrow 2^-} \frac{|x - 2|}{x - 2} = \lim_{x \rightarrow 2^-} \frac{-(x - 2)}{x - 2} = \lim_{x \rightarrow 2^-} -1 = -1.$$

For the righthand limit (where $x > 2$), we can write $|x - 2| = x - 2$, giving

$$\lim_{x \rightarrow 2^+} \frac{|x - 2|}{x - 2} = \lim_{x \rightarrow 2^+} \frac{x - 2}{x - 2} = \lim_{x \rightarrow 2^+} 1 = 1.$$

Since the one-sided limits are not equal, we can conclude that $\lim_{x \rightarrow 2} \frac{|x - 2|}{x - 2}$ does not exist.

3. (a) Although $f(x)$ is a piecewise function, its definition does not change at $x = \frac{\pi}{6}$, so we can use direct substitution. Since $f(x) = \cos(x)$ for all $x \leq 0$,

$$\lim_{x \rightarrow -\frac{\pi}{6}} f(x) = \lim_{x \rightarrow -\frac{\pi}{6}} \cos(x) = \cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}.$$

- (b) Since $f(x)$ changes its definition at $x = 0$, we must consider the one-sided limits. Immediately to the left of $x = 0$, $f(x) = \cos(x)$ so

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \cos(x) = \cos(0) = 1.$$

Immediately to the right of $x = 0$, $f(x) = 1 - 4x$ so

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} (1 - 4x) = 1 - 4(0) = 1.$$

Since these are in agreement,

$$\lim_{x \rightarrow 0} f(x) = 1.$$

- (c) Again, because $f(x)$ changes its definition at $x = 3$, we must calculate the one-sided limits. Immediately to the left of $x = 3$, $f(x) = 1 - 4x$ so

$$\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} (1 - 4x) = 1 - 4(3) = -11.$$

Immediately to the right of $x = 3$, $f(x) = \frac{9}{x}$ so

$$\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} \frac{9}{x} = \frac{9}{3} = 3.$$

Since the one-sided limits are not equal, $\lim_{x \rightarrow 3} f(x)$ does not exist.

4. Since this is a piecewise function whose behaviour changes at $x = -2$, we must check the one-sided limits:

$$\lim_{x \rightarrow -2^-} f(x) = \lim_{x \rightarrow -2^-} k^2x = -2k^2$$

and

$$\lim_{x \rightarrow -2^+} f(x) = \lim_{x \rightarrow -2^+} (4k - x) = 4k + 2.$$

If the limit exists, then these one-sided limits must be equal, so we set

$$-2k^2 = 4k + 2$$

$$2k^2 + 4k + 2 = 0$$

$$2(k + 1)^2 = 0,$$

and hence $k = -1$.

(Note that the second part of the definition of $f(x)$ did not affect our workings because it applies only for $x = -2$, which has no effect on the limit as x approaches -2 .)