MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 8 **MATHEMATICS 1000** FALL 2024

Due: Friday, November 22nd, 2024 at 11:59pm. See the Gradescope Handout for submission information.

> **Note:** You should complete the WeBWorK problem sets "Higher Derivatives" and "Extrema and Inflection Points" as well as Worksheets 3.7, 4.1 and 4.2, before you work on this assignment.

1. Find $\dfrac{d^2y}{dx^2}$ given $y = 2x - y^2$.

2. An ice sculpture in the shape of an inverted cone is melting such that its height shrinks at a rate of $\frac{1}{3}$ metres per hour but its radius remains constant. How fast is the cone's volume decreasing when its height is 1 metre and its volume is 3π cubic metres?

3. A woman walks in a straight line across the edge of a stage at a speed of 3 feet per second. A spotlight positioned 60 feet from the stage is kept focussed on the woman. Determine the rate at which the spotlight is rotating at the moment when the woman is 25 feet from point on the stage closest to the spotlight.

4. Consider the function $f(x) = \dfrac{x^2(x+2)}{(x-1)^3}$ with derivatives

$$f'(x) = \frac{-x(5x+4)}{(x-1)^4} \quad \text{and} \quad f''(x) = \frac{2(5x+1)(x+2)}{(x-1)^5}.$$

   (a) Determine the intervals on which $f(x)$ is increasing or decreasing, and identify any relative extrema.

   (b) Determine the intervals on which $f(x)$ is concave upward or concave downward, and identify any points of inflection.