Les comptes-rendus de livres présentent aux lecteurs de la SMC des ouvrages intéressants sur les mathématiques et l’enseignement des mathématiques dans un large éventail de domaines et sous-domaines. Vos commentaires, suggestions et propositions sont les bienvenus.

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**Numerical Linear Algebra: An Introduction**

by Holger Wendland

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Large systems of linear algebraic equations and associated eigenvalue problems are prevalent in many areas in applied mathematics, arising naturally, for example, from the discretization of continuous problems. Holger Wendland is a professor and chair in the Department of Mathematics in the Faculty of Mathematics, Physics and Computer Sciences at the Universität Bayreuth in Germany. He is a highly cited researcher in numerical analysis and numerical linear algebra and serves on the editorial board of several prestigious journals including the SIAM Journal on Numerical Analysis and Numerical Algorithms. Professor Wendland provides a relatively concise, yet self-contained, introduction to the design, analysis, and implementation of traditional numerical algorithms to solve these linear problems on a computer. He motivates the topic with a sequence of diverse applications: interpolation, the numerical solution of boundary value problems and integral equations. Yet, the author manages, in just under 400 pages, to also introduce state of the art additional topics including preconditioning, multilevel expansions, domain decomposition and applications to compressed sensing, that often do not appear in existing texts. In this way, the book finds a niche between the classic reference text “Matrix Computations” by Golub & Van Loan, which provides a somewhat terse encyclopedic treatise on the topic, and more specialized texts on iterative methods (Varga, 1962 or Saad, 2003), multgrid methods (Briggs et al., 2000 or Trottenberg, 2000) or domain decomposition methods (Toselli & Widlund, 2005 or Donellan et al., 2015).

The material could comprise a second numerical analysis course at the senior undergraduate level or an entry-level graduate course. Although a brief review of the basics of linear algebra is provided in the first chapter, students are expected to have already completed a standard undergraduate linear algebra class. As the author indicates in the preface, an essential part of a course taught with this book is the implementation of the algorithms in a programming language of the student’s (or instructor’s) choice. Pseudocode for all algorithms is given to assist.

A mathematically rigorous, formal style (Definitions, Theorems, Proofs) is kept throughout the text, even in the advanced topics and application sections, while keeping the writing relatively light. Although brief, each application section and additional topic section clearly describes the problem of interest, or the technique, providing the essential pieces which will motivate keen students (and researchers!) to follow up with dedicated introductory texts.

Each chapter ends with a set of exercises which generally are of a theoretical nature. To motivate the reader to implement the algorithms, it may have been useful to provide exercises of a numerical nature.

Most of the first six chapters of Wendland’s text is not dissimilar, in scope and detail, to Trefethen and Bau’s book “Numerical linear algebra” (SIAM 1997). The book under review distinguishes itself in the latter part of Chapter 6 and in Chapters 7-9 of Part III — which comprises more than half of the book’s pages. The topics in these chapters are generally not seen in books at this level. The material would probably be considered graduate level, but Wendland’s style makes it accessible to a more advanced undergraduate student who has worked through Parts I and II of the text.

The last section of Chapter 6 gives an introduction to multigrid methods which are fast, sometimes optimal, methods, to solve problems defined on a sequence of grids. Chapter 7 discusses three situations (fast multipole methods, hierarchical matrices and domain decomposition) which give more efficient matrix-vector products, hence potentially reducing the cost of iterative methods - even in situations where the matrix does not enjoy sparsity. The idea of improving the convergence of iterative methods is continued in Chapter 8 with a discussion of various preconditioning strategies which attempt to replace the linear system by a better conditioned, equivalent, system.

The final chapter considers underdetermined linear systems which generally have an infinite number of solutions. Compressed sensing is a technique which recovers uniqueness by imposing a sparsity constraint on the solution. Applications of this approach are numerous and include facial recognition and magnetic resonance imaging.

In summary, this text is a welcome addition to a numerical linear algebra bookshelf, either as a solid text to use in a classroom situation, or a very readable introduction for a new graduate student.