Assignment 1, Due Date Sep 18

Read the textbook. Please answer the following questions. 1. Briefly explain the minus sign:

(a) in conservation law $\frac{\partial e}{\partial t} = -\frac{\partial \phi}{\partial x}$;

(b) in Fourier's law $\phi = -K_0 \frac{\partial u}{\partial x}$;

2. Derive the heat equation for a rod assuming constant thermal properties and no sources.

(a) Consider the total thermal energy between x and $x + \Delta x$;

(b) Consider the total thermal energy between $x = a$ and $x = b$;

3. Two one-dimensional rods of differential materials joined at $x = x_0$ are said to be in perfect thermal contact if the temperature is continuous at $x = x_0$:

$$u(x_{0-}, t) = u(x_{0+}, t)$$

and no heat energy is lost at $x = x_0$ (i.e., the heat energy flowing out of one flows into the other). What mathematical equation represents the latter condition at $x = x_0$? Under what condition is $\partial u/\partial x$ continuous at $x = x_0$?

4. Determine the equilibrium temperature distribution for one dimensional rod with constant thermal properties with the following sources and boundary conditions:

(a)

$$Q = 0,\ u(0) = T,\ u(L) = 0;$$

(b)

$$\frac{Q}{K_0} = 1,\ \ u(0) = T_1,\ u(L) = T_2;$$

(c)

$$Q = 0,\ u(0) = T,\ \frac{\partial u}{\partial x}(L) + u(L) = 0.$$

5. Determine the equilibrium temperature distribution for one dimentional rod composed of two different materials in perfect thermal contact at $x = 1$. For $0 < x < 1$, there is one material $(c\rho = 1, K_0 = 1)$ with a constant source $(Q = 1)$, whereas for the other $1 < x < 2$ there are no sources $(Q = 0, c\rho = 1, K_0 = 2)$ with $u(0) = 0$ and $u(2) = 0$.

6. The two ends of a uniform rod of length L are insulated. There is a constant source of thermal energy $Q_0 \neq 0$, and the temperature is initially $u(x, 0) = f(x)$.

(a) Show mathematically that there does not exist any equilibrium temperature distribution. Briefly explain physically.

(b) Calculate the total thermal energy in the entire rod.

7. For the following problem, determine an equilibrium temperature distribution (if exists). For what values of β are there solutions?

(a)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2},\ \ u(x,0) = f(x),\ \ \frac{\partial u}{\partial x}(0,t) = 1,\ \ \frac{\partial u}{\partial x}(L,t) = \beta;$$

(b)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + x - \beta,\ u(x,0) = f(x),\ \ \frac{\partial u}{\partial x}(0,t) = 0,\ \ \frac{\partial u}{\partial x}(L,t) = 0.$$